

THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALLS

UNIVERSITY OF LONDON

291 0001 ZA

Diploma Examination
for External Students

**COMPUTING AND INFORMATION SYSTEMS AND
CREATIVE COMPUTING**

Mathematics for Business

Dateline: Tuesday 5 May 2009 : 10.00 – 1.00 pm

Duration: 3 hours

There are ten questions in this paper. Candidates should answer **TEN** questions. Full marks will be awarded for complete answers to **TEN** questions.

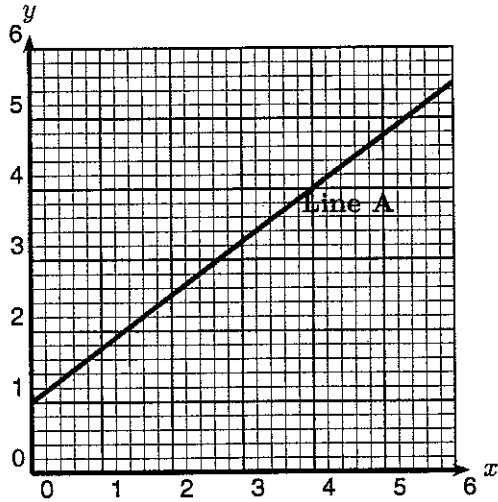
A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, text or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

Graph paper is required for this examination.

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- 1 (a) Find the equation of line A shown on the graph below. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

[3]



- (b) What are the co-ordinates of the point where line A intersects the line $x = 20$?

[2]

- (c) Write down the equation of line B which is parallel to line A and crosses the y -axis at the point $(0, 4)$.

[2]

- (d) Is the shortest distance between line A and line B:

- less than 3 units,
- equal to 3 units,
- greater than 3 units?

Justify your answer.

[3]

2. A bakery makes cakes which are sold to a supermarket for \$2 each. The bakery has fixed costs of \$100 per day plus variable costs of \$1.40 per cake.
- (a) How much profit does the bakery make in a day if it produces and sells 1,000 cakes?
[2]
- (b) Write an equation expressing profit in terms of n where n is the number of cakes produced.
[1]
- (c) How many cakes does the bakery have to make and sell each day in order to make any profit?
[2]
- (d) The price of ingredients increases resulting in the variable costs per cake increasing by 10%. The fixed costs of the bakery rise by 5%. If the bakery wants to make the same profit for 1,000 cakes as it did before the price increases, how much should it charge the supermarket for each cake?
[5]
3. (a) Put brackets into the following expressions to make them correct.
- i. $4 \times 9 - 2 + 6 = 34$
ii. $x^2 - x - 3 \times x + 3 = 9$
[2]
- (b) Find the **two** sets of solutions to the simultaneous equations:
- $$\begin{aligned}x + y &= 4 \\x + y^2 &= 16\end{aligned}$$
- [4]
- (c) Tickets for a show cost either \$32 or \$45. Ticket sales for a particular show totalled \$8,810. There were 242 people in the audience. How many \$45 tickets were sold?
[4]
4. (a) Find all of the solutions of the equation
- $$2x^3 - 6x^2 + 4x = 0$$
- [3]
- (b) Use differentiation to determine the turning points of the function
- $$f(x) = 2x^3 - 6x^2 + 4x$$
- Give your answers correct to two decimal places.
[4]
- (c) Draw a sketch of the function $f(x) = 2x^3 - 6x^2 + 4x$ clearly showing the turning points and points of intersection with the axis.
[3]

5. (a) Given the matrices

$$\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7 & 4 \\ -1 & 1 \\ 2 & -3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

For each of the following either compute the given matrix or say that it is undefined.

- i. ABC
- ii. CBA
- iii. AI_2
- iv. $C^T + B$
- v. $C^T B$

[7]

- (b) Write the system of simultaneous equations that are represented by the following augmented matrix:

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 19 \\ -1 & 2 & 1 & 0 \\ -2 & -2 & 2 & 12 \end{array} \right)$$

[3]

6. A company produces a fruit juice drink in tropical and exotic varieties. The tropical drink requires 600ml of orange juice and 100ml of mango juice per litre plus other secret ingredients. The exotic drink requires 400ml of orange juice and 300ml of mango juice per litre plus other secret ingredients. The exotic drink sells at a profit of \$1.3 per litre and the tropical drink sells at a profit of \$1.2 per litre. In one week, the company has 600 litres of orange juice and 200 litres of mango juice (plus unlimited quantities of the secret ingredient).

- (a) Formulate the problem of finding how many litres of tropical drink and how many litres of exotic drink the company should make with the given ingredients in order to maximise their profit as a linear programming problem in two variables.

[4]

- (b) Draw a sketch graph to show the feasible region for this problem. List the corners of the feasible region.

[4]

- (c) Determine the number of litres of tropical juice and exotic drink that the company should produce in order to maximise their profit and find this profit.

[2]

7. (a) The function $f(x)$ is given by two different formulae as follows:

$$f(x) = \begin{cases} x & \text{if } x \text{ to the nearest integer is even} \\ -x & \text{if } x \text{ to the nearest integer is odd} \end{cases}$$

- i. Draw a sketch of the function $f(x)$ for $-4 \leq x \leq 4$.

[4]

- ii. Can $f(x)$ be differentiated at every point, some points, or no points? Justify your answer.

[2]

- (b) The function $g(x)$ is given by

$$g(x) = \frac{2x^2 + 6}{x + 1}$$

Show that

$$g'(x) = \frac{2(x + 3)(x - 1)}{(x + 1)^2}$$

[4]

8. (a) Write down the profit function Π , in terms of quantity Q , given the total revenue function:

$$TR = 2Q^2 + 74Q$$

and the total cost function:

$$TC = Q^3 - 28Q^2 + 182Q + 340$$

[2]

- (b) Find all the values of Q which give the relative extrema and points of inflexion for the function $\Pi(Q)$ and classify these points.

[6]

- (c) What is the maximum profit?

[2]

9. Solve the following equations giving your answers correct to three decimal places where appropriate.

(a) $8^x = 5$ [1]

(b) $\log_3 x^5 = 10$ [2]

(c) $\log_x 4 + \log_x 9 = 2$ [2]

(d) $e^x = 10$ [1]

(e) $3^{2x} - 3^{x+2} = 90$ (Hint: use the substitution $z = 3^x$) [4]

10. The graph of function $f(x)$ cuts the x -axis at points $(-1, 0)$, $(2, 0)$ and $(3, 0)$ and the y -axis at the point $(0, 6)$. The graph of $f(x)$ does not cut the axes at any other points.

(a) By considering the roots or otherwise, find the equation of the function $f(x)$ and draw a sketch of the graph [5]

(b) Find the total area enclosed by the curve $f(x)$ and the x -axis. [5]

END OF EXAMINATION