

THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALLS

UNIVERSITY OF LONDON

291 0102 ZB

BSc/Diploma Examination
for External Students

**COMPUTING AND INFORMATION SYSTEMS AND
CREATIVE COMPUTING**

Mathematics for Computing

Dateline: Monday 11 May 2009 : 10.00 – 1.00 pm

Duration: 3 hours

There are ten questions in this paper. Candidates should answer **TEN** questions. Full marks will be awarded for complete answers to **TEN** questions.

A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, text or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

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UL09/875

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Question 1

- (a) What hexadecimal number has to be added to $(A97)_{16}$ to obtain $(D03)_{16}$? [2]
- (b) Write the decimal number 249 in
 - (i) base 2
 - (ii) base 16. [3]
- (c) (i) Express the fraction $\frac{9}{11}$ as a recurring decimal.
(ii) Express the recurring decimal $0.1717\dots$ as a fraction in its lowest terms. [4]
- (d) Give an example of (i) a rational number and (ii) an irrational number. [1]

Question 2

- (a) (i) Describe the set A by the listing method giving the first three elements and last element of A where $A = \{4r - 1 : r \in \mathbb{Z}^+ \text{ and } -1 \leq r \leq 50\}$.
(ii) Describe the set B by the rule of inclusion method where $B = \{4, 16, 64, \dots, 4096\}$. [4]
- (b) (i) Write out and complete the following table:

A	B	C	$B \cap C$	$(B \cap C)'$	$(B \cap C)' - A$	X
0	0	0				0
0	0	1				0
0	1	0				0
0	1	1				1
1	0	0				1
1	0	1				1
1	1	0				1
1	1	1				1

- (ii) Draw a labelled Venn diagram showing A , B and C intersecting in the most general way and shade the region X on it.
- (iii) Find an expression which defines the set X in terms of A , B and C and set operations. [6]

Question 3 (a) Let $n \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p, q be the following propositions concerning the integer n .

$$p : n \text{ is odd} \quad q : n < 5.$$

By drawing up the appropriate truth table find the truth set for each of the propositions $p \vee \neg q$; $\neg(q \rightarrow p)$. [4]

- (b) (i) Construct and draw a logic network that accepts as inputs p and q , which may independently have the value 0 or 1, and gives as final output $(p \wedge q) \vee \neg q$. Label all the gates appropriately and also give labels to show the output from each gate.
- (ii) Construct a logic table to show the value of the output corresponding to each combination of values (0 or 1) for the inputs p and q .
- (iii) Show that $(p \wedge q) \vee \neg q$ is equivalent to $q \rightarrow p$. [6]

Question 4 (a) What properties should a graph possess if it is

- (i) simple (ii) connected? [2]

(b) Given a graph G with degree sequence

$$4, 3, 3, 3, 2, 1.$$

- (i) How many vertices are there in G ?
- (ii) Find the number of edges in G , explaining how you obtain your answer.
- (iii) Draw an example of a simple graph G with the degree sequence

$$4, 3, 3, 3, 2, 1.$$

[6]

(c) (i) Say why it is not possible to construct a simple graph with the degree sequence

$$4, 2, 2, 2.$$

- (ii) Show it is possible to construct a graph with this degree sequence if we do not require it to be simple. [2]

Question 5

- (a) Given the ceiling function: $\lceil x \rceil = n$ where $n - 1 < x \leq n$, $n \in \mathbb{Z}$.
Let $A = \{1, 2, 3, 4, 5\}$ and $f(x) = \left\lceil \frac{x^2 - 1}{4} \right\rceil$ where $f : A \rightarrow \mathbb{Z}$.
- (i) Find $f(2)$ and the ancestor of 0.
 - (ii) Find the range of f .
 - (iii) Is f invertible? Justify your answer. [3]
- (b) Let $g(n) = \left\lceil \frac{n-1}{4} \right\rceil$ where $g : \mathbb{Z} \rightarrow \mathbb{Z}$.
- (i) Find $g(4)$ and the set of ancestors of 0.
 - (ii) Find the range of g .
 - (iii) Is g invertible? Justify your answer. [4]
- (c) Let $L = \{a, b, c\}$, $M = \{1, 2, 3, 4\}$ and $N = \{x, y\}$. Draw arrow diagrams for the following functions:
- (i) a function from $L \rightarrow M$ that is one to one but not onto;
 - (ii) a function from $L \rightarrow N$ that is onto but not one to one;
 - (iii) a function from $L \rightarrow L$ that is both one to one and onto. [3]

Question 6 Let n be a positive integer.

- (a) Given $s_n = 1 + 2 + 3 + 4 + \dots + n$
- (i) Calculate s_1 , s_2 and s_3 .
 - (ii) Give a recurrence relation which expresses s_{n+1} in terms of s_n for all $n \geq 1$.
 - (iii) It can be proved by induction that $s_n = \frac{n(n+1)}{2}$. Use this result to find the sum of the first 300 positive integers. [4]
- (b) Write the following two expressions in \sum notation with appropriate limits and use (a)(iii) to calculate each sum:
- (i) $2 + 4 + 6 + 8 + \dots + 1000$
 - (ii) $4 + 7 + \dots + ((3 \times 1000) + 2)$. [6]

Question 7

- (a) Given the set $S = \{a, b, c, d\}$.
- (i) Describe briefly how each subset of S can be represented by a unique 4 digit binary string.
 - (ii) Write down the string corresponding to the subset $\{a, c, d\}$ and the subset corresponding to the string 1011.
 - (iii) What is the total number of subsets of S ? [4]
- (b) T is the set $\{\{a, b\}, \{a\}, \{b\}, \{a, b, d\}\}$. R is a relation defined on T as follows:

xRy if $X \subseteq Y$ where X and Y are elements of T .

Draw the relationship digraph for R on T and say, with reason, whether this relation is

- (i) reflexive
- (ii) symmetric
- (iii) transitive
- (iv) a partial order. [6]

Question 8 (a) Determine the number of different 3-digit strings using only digits from the set $\{1, 2, 3, 4, 5, 6, 7\}$ where repetitions are allowed. How many of these strings will have all their digits distinct? [2]

- (b) A deck of cards contains six cards numbered 1, 2, 3, 4, 5, 6 and 7. An experiment is carried out in which three cards are chosen from this deck **without** replacement and the result is recorded as an ordered triple, such as (1,2,4), where this result is different from the result (2,4,1).
- (i) Let A be the event that the first card is odd and B the event that the last card is a 7. Calculate the number of elements in each of the sets A , B , $A \cap B$ and $A \cup B$.
 - (ii) Hence calculate the probabilities of A , B , $A \cap B$ and $A \cup B$. [8]

Question 9

- (a) What two properties must a graph satisfy in order to be a tree? [2]
- (b) Let H be a subgraph of a graph G . Explain what it means to say that H is a spanning tree of G . [1]
- (c) Let G be the graph with the following adjacency list
- $a : b,$
 - $b : a, c, e$
 - $c : b, d$
 - $d : c, e$
 - $e : b, d, f$
 - $f : e.$
- (i) Draw this graph, G .
- (ii) Find and draw all spanning trees of G .
- (iii) How many non-isomorphic spanning trees does G have? [4]
- (d) A binary search tree is designed to store an ordered list of 6000 records, numbered 1,2,3,...,6000 at its internal nodes.
- (i) Draw levels 0, 1 and 2 of this tree, showing which number record is stored at the root and at each of the nodes at level 1 and 2, making it clear which records are at each level.
- (ii) What is the height of this tree? [3]

Question 10

- (a) Given the following matrices A and B and C where

$$\mathbf{A} = \begin{pmatrix} -4 & 3 \\ 2 & 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 & -2 \\ 1 & 3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -4 & 0 & 5 \\ 1 & -3 & 2 \end{pmatrix}$$

- (i) Calculate $2\mathbf{BC}$.
- (ii) Calculate $(\mathbf{A} + \mathbf{B})\mathbf{C}$. [4]
- (b) (i) Write down the augmented matrix for the following system of equations.

$$\begin{aligned} 2x + y - z &= 1 \\ x - y + z &= 2 \\ x + 2y + 2z &= 11 \end{aligned}$$

- (ii) Use Gaussian elimination to solve the system. [6]

END OF EXAMINATION