THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALLS

UNIVERSITY OF LONDON **291 0311 ZA**

BSc Examination
for External Students

COMPUTING AND INFORMATION SYSTEMS AND CREATIVE COMPUTING

Neural Networks

Dateline: Thursday 26 May 2009 : 2.30 – 4.45 pm

Duration: 2 hours 15 minutes

Full marks will be awarded for complete answers to **FOUR** questions. Candidates should not attempt more than **FOUR** questions on this paper.

A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, texts or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

Question 1.

a) How many layers in a network are sufficient to classify an n-dimensional point into a polygon? Define the notion of a polygon. **[4]**

b) Explain whether we can design networks in which the neurons use different activation functions. **[4]**

c) Describe whether the value-descending strategy for Boltzmann training of multilayer networks can reach a solution with zero error using the notions of local and global minimum. **[4]**

d) Give the formula for accepting weight changes in the Boltzmann training algorithm for multilayer networks. **[5]**

e) Assume a single discrete neuron with activation function: *Threshold* $(s) = 1$ if $s > 0$ and -1 otherwise. This neuron should be trained with the following algorithm:

Repeat for each *training example* (x_e, y_e)
- *calculate* the output: $o_e = Threshold(\Sigma_i w_i x_{ie})$
- if $(y_T = o_e)$

{
$w_i = w_i - x_{ie}$ when $o_e = -1$
$w_i = w_i + x_{ie}$ when $o_e = 1$
}

until termination condition is satisfied.

Consider the following initial weights: $(w_1, w_2) = (0, -1)$, and the following examples:

x_1	x_2	y_T
1	−1	−1
−0.5	0.5	1

Demonstrate how the weights are updated after each example in incremental manner. **[8]**

Question 2.

a) Consider a single neuron with two inputs x_1 and x_2. This single neuron uses a discrete activation function. The weights on the connections are: $w_0 = 2$, $w_1 = -1$, and $w_2 = -1$.

i) Write the equation of the line modelled by this neuron. **[2]**

ii) Plot in the two-dimensional space the line modelled by this neuron. **[4]**

iii) Identify in which half of this plane are the points corresponding to negative outputs, and in which half are the points corresponding to positive outputs. **[4]**

b) Perform training of a single discrete neuron with two inputs x_1, x_2, and bias x_3=1, using training value η= 0.2, initial weights: $(w_1, w_2, w_3) = (-1, 0, 1)$, and the following examples:

x_1	x_2	x_3	y_1
-2	1	1	0
1	2	1	1
1	0	1	0
0	1	1	0
1	1	1	0

Give the actual neuron output, and the updated weights after each example. **[15]**

Question 3.

a) When is the neural network training process unsupervised? **[3]**

b) Which of the following Boolean functions, AND, OR, XOR and NOT, cannot be modelled by a single neuron with two inputs. **[4]**

c) Explain how the performance of a single neuron with two inputs and a discrete activation function will be affected when the weights and the threshold are multiplied by a positive constant? **[4]**

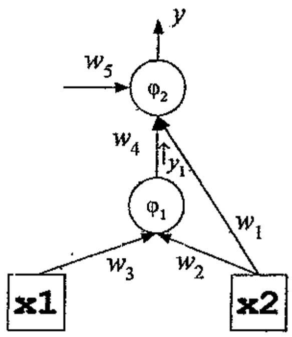
d) Let a sigmoidal neuron with two inputs x_1, x_2, and bias $x_3=1$ be given. Train this neuron with the Widrow-Hoff rule using training value $\eta = 0.3$, initial weights: $(w_1, w_2, w_3) = (0.2, 0.1, -0.3)$, and the following training vectors:

x_1	x_2	x_3	y_T
−1.5	1.2	1	1
1.1	−1.3	1	0

Show the neuron output and the weight update after each example. **[14]**

Question 4.

Assume a multilayer feedforward neural network with one hidden neuron, one output neuron, and two inputs (x_1, x_2) as shown in the figure below. The neurons use sigmoidal activation functions. The weights on the connections toward the hidden neuron are w_2, and w_3. The weights on the connections toward the output neuron are w_1, w_4, and w_5 (bias). Train this multilayer network with the backpropagation algorithm using learning rate η, and target d.



i) Develop the equations for computing the hidden node output y_1 and the overall network output y during the forward propagation step. **[4]**

ii) Develop the equations for computing the changes to weights Δw_1, Δw_4 and Δw_5 on the connections toward the output node. **[9]**

iii) Give the equations for computing the weight changes Δw_2 and Δw_3 on the connections toward the hidden node. **[9]**

iv) Give the formula for weight updating with momentum for backpropagation training of multilayer networks. **[3]**

Question 5.

a) Explain which state in Hopfield neural networks is called energy well. Discuss whether the training process in Hopfield networks can reach an energy well starting from any initial state. **[6]**

b) Write the formula for computing the energy level in Hopfield neural networks. **[4]**

c) Train a Hopfield network with three ($n = 3$) neurons: N_1, N_2, N_3, and a neuron N_0 whose output is always $x_0 = 1$. Consider the following training input pattern: $[\, x_1, x_2, x_3 \,] = [\, 1 \; 1 \; 0 \,]$, negative target output, and the following initial weight matrix:

$$\mathbf{W} = \begin{matrix} 0 & -0.3 & -0.4 & -0.1 \\ -0.3 & 0 & 0.1 & 0.3 \\ -0.4 & 0.1 & 0 & 0.1 \\ -0.1 & 0.3 & 0.1 & 0 \end{matrix}$$

i) Update the weights of the first neuron using the Widrow-Hoff rule. **[5]**

ii) Show that the updated network is unstable for the second neuron N_2 and retrain it. **[5]**

iii) Show that the updated network is unstable for the third neuron N_3 and retrain it. **[5]**

Question 6.

a) What is the role of Kohonen networks as unsupervised learning tools? **[3]**

b) Which are the two alternative ways of initialisation of the weights in the Kohonen layer of counterpropagation networks? Explain your answer. **[5]**

c) Give the normalisation formula used in Kohonen networks. **[3]**

d) Consider a Kohonen network having two neurons with four inputs each. Demonstrate training of this network using the following initial weights $\mathbf{w}_1 = (0.1, 0.4, -0.3, -0.2)$, $\mathbf{w}_2 = (0.2, -0.1, -0.4, 0.3)$, and the input vector: $\mathbf{x} = (0.1, -0.3, 0.1, -0.2)$.

i) Compute the output of each neuron. **[4]**

ii) Decide which is the winning neuron to fire. **[2]**

iii) Train the weights of the winning neuron using learning value $\eta = 0.3$. **[8]**

END OF EXAMINATION