

THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALLS

UNIVERSITY OF LONDON

291 0311 ZB

**BSc Examination**  
for External Students

**COMPUTING AND INFORMATION SYSTEMS AND  
CREATIVE COMPUTING**

**Neural Networks**

Dateline: Thursday 26 May 2009 : 2.30 – 4.45 pm

Duration: 2 hours 15 minutes

Full marks will be awarded for complete answers to **FOUR** questions. Candidates should not attempt more than **FOUR** questions on this paper.

A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, texts or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

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**Question 1.**

- a) How many layers should be used in a network to classify a point into a concave polytope? What exactly does concave polytope mean? [4]
- b) Explain briefly if there is some delay between the firing of the neurons in the first and second layers in multilayer networks. [4]
- c) What is the implication of using the value-descending strategy for Boltzmann training of multilayer neural networks? [4]
- d) Give the formula for computing the error necessary to evaluate the performance of a multilayer network trained by the Boltzmann algorithm. [5]
- e) Consider a single discrete neuron with activation function:  $Threshold(s) = 1$  if  $s > 0$  and  $-1$  otherwise. This neuron should be trained with the following algorithm:
- ```
Repeat for each training example  $(x_e, y_e)$ 
- calculate the output:  $o_e = Threshold(\sum_i w_i x_{ie})$ 
- if  $(y_e \neq o_e)$  // if there is an error
  {
     $w_i = w_i - x_{ie}$  when  $o_e = -1$ 
     $w_i = w_i + x_{ie}$  when  $o_e = 1$ 
  }
until termination condition is satisfied.
```

Use initial weights:  $(w_1, w_2) = (0, 0)$ , and the following examples:

| $x_1$ | $x_2$ | $y$ |
|-------|-------|-----|
| 0.5   | -0.5  | 1   |
| -1    | 1     | -1  |

Show how the weights are updated after each example in an incremental manner. [8]

**Question 2.**

a) Let a single neuron with two inputs:  $x_1, x_2$ , and a discrete activation function be given. The connection weights are:  $w_1 = 2, w_2 = -2$ , and  $w_0 = 0$ .

- i) Give the equation of the line modelled by this neuron. [2]
- ii) Draw the line modelled by this neuron in the two-dimensional plane. [4]
- iii) Show in which half of this plane the points corresponding to positive outputs are, and in which half the points corresponding to negative outputs are. [4]

b) Train a single discrete neuron with two inputs  $x_1, x_2$ , and bias  $x_3=1$ , using initial weights:  $(w_1, w_2, w_3) = (1, -1, 0)$ , training value  $\eta = 0.2$  and the following examples:

| $x_1$ | $x_2$ | $x_3$ | $y_T$ |
|-------|-------|-------|-------|
| 1     | -2    | 1     | 0     |
| 1     | 1     | 1     | 0     |
| 0     | 2     | 1     | 1     |
| 2     | 1     | 1     | 0     |
| 1     | 0     | 1     | 1     |

Show the actual neuron output, and the updated weights after each example. [15]

**Question 3.**

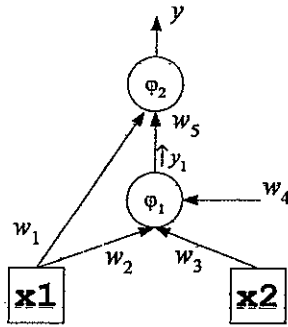
- a) Which network training process is supervised? [3]
- b) Will there be a change in the performance of a single neuron with two inputs and a discrete activation function if the weights and the threshold are multiplied by a negative constant? [4]
- c) Explain briefly which of the Boolean functions, AND, OR, XOR and NOT, can be modelled by a single neuron with two inputs. [4]
- d) Apply the Widrow-Hoff rule for sigmoidal neurons to train a sigmoidal neuron with two inputs  $x_1$ ,  $x_2$ , and bias  $x_3=1$ . Assume that the training value is  $\eta = 0.2$ , the initial weights are:  $(w_1, w_2, w_3) = (0.1, -0.2, 0.3)$ , and the training vectors are:

| $x_1$ | $x_2$ | $x_3$ | $y_T$ |
|-------|-------|-------|-------|
| 1.2   | 0.7   | 1     | 0     |
| -1.3  | 1.5   | 1     | 1     |

Compute the neuron output and the weight update after each example. [14]

**Question 4.**

Consider a multilayer feedforward neural network with one hidden neuron and one output neuron. This network has two inputs ( $x_1, x_2$ ). Both neurons use sigmoidal activation functions. The connections between the nodes are illustrated in the figure below: the weights on the connections toward the hidden neuron are  $w_2, w_3$  and  $w_4$  (bias) and the weights on the connections toward the output neuron are:  $w_1$  and  $w_5$ . This multilayer network has to be trained with the backpropagation algorithm using learning rate  $\eta$ , and target  $d$ .



- i) Give the equations for computing the hidden node output  $y_1$  and the overall network output  $y$  during the forward propagation step. [4]
- ii) Give the equations for computing the weight changes  $\Delta w_1$  and  $\Delta w_5$  on the connections toward the output node during backpropagation. [6]
- iii) Give the equations for computing the weight changes  $\Delta w_2, \Delta w_3$  and  $\Delta w_4$  on the connections toward the hidden node during backpropagation. [12]
- iv) How do we compute the momentum term in the formula for backpropagation training of multilayer networks? [3]

**Question 5.**

- a) Which is the most distinguishing characteristic of Hopfield neural networks with respect to the connectivity pattern? Explain your answer. [6]
- b) Write the formula for updating of the state in Hopfield neural networks. [4]
- c) This question is concerned with demonstrating training of a Hopfield network with three ( $n = 3$ ) neurons:  $N_1, N_2, N_3$ , and a neuron  $N_0$  whose output is always  $x_0 = 1$ . Consider the following training input pattern:  $[x_1, x_2, x_3] = [0 \ 1 \ 1]$ , negative target output, and the following initial weight matrix:

$$\mathbf{W} = \begin{matrix} & 0 & 0.1 & -0.4 & -0.5 \\ 0.1 & 0 & -0.1 & 0.2 \\ -0.4 & -0.1 & 0 & 0.2 \\ -0.5 & 0.2 & 0.2 & 0 \end{matrix}$$

- i) Use the Widrow-Hoff rule to update the first neuron weights. [5]
- ii) Demonstrate that the updated network is unstable for the second neuron  $N_2$  and retrain it. [5]
- iii) Demonstrate that the updated network is unstable for the third neuron  $N_3$  and retrain it. [5]

**Question 6.**

- a) Explain how the number of neurons in the Kohonen layer of counterpropagation networks impacts their initialisation. [5]
- b) How do we train Kohonen networks: in supervised or unsupervised mode? [3]
- c) Give the normalisation formula used in Kohonen networks. [3]
- d) Let a Kohonen network with two neurons having four inputs each be given. Train this network using initial weights  $\mathbf{w}_1 = (0.5, -0.4, 0.3, 0.2)$  and  $\mathbf{w}_2 = (0.2, -0.3, -0.4, 0.1)$ , and the following input vector:  $\mathbf{x} = (-0.1, 0.3, 0.2, 0.1)$ .
  - i) Calculate the output of each neuron. [4]
  - ii) Determine the index of the winning neuron. [2]
  - iii) Update the weights of the winning neuron using learning value  $\eta = 0.15$ . [8]

**END OF EXAMINATION**