THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALLS

UNIVERSITY OF LONDON **291 0325 ZB**

BSc Examination
for External Students

COMPUTING AND INFORMATION SYSTEMS AND CREATIVE COMPUTING

Data Compression

Dateline: Friday 22 May 2009 : 2.30 – 4.45 pm

Duration: 2 hours 15 minutes

Candidates should answer **THREE** questions only. Full marks will be awarded for complete answers to **THREE** questions. There are 75 marks available on this paper.

A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, texts or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

Question 1

(a) Consider a binary source $S_1 = (A, B)$, where A and B occur independently, and the extended alphabet $S_2 = (AA, AB, BA, BB)$. [10]

(i) Prove that the (first-order) entropy of S_2 is double the value of the entropy of S_1

(ii) Suppose that $S_1 = (A, B) = (0.1, 0.9)$. Demonstrate the entropy relationship between S_1 and S_2. Show all your work.

(b) Outline in a flowchart the adaptive Huffman algorithm for decoding. [5]

(c) Demonstrate how your adaptive Huffman algorithm works for decoding `10100001010100000111101001000 1`. Trace the values of the `input`, `output`, `alphabet` and the `tree structure` on each step. Write down the decoding result and compute the compression ratio. Assume that the fixed-length codes for characters $A, \cdots, Z$, are $01000001, \cdots, 01011010$, respectively, and that each new symbol is added to the front (left most position) of the alphabet before a stable sort. [10]

Question 2

(a) What is a (binary) prefix code? Give an example of a prefix code and another example of a non-prefix code that is uniquely decodable. Explain your examples. [5]

(b) Explain why there is no lossless compression program that can compress all the files even by one byte. Justify your answer. [5]

(c) Explain the main ideas of *progressive image compression*. What are the main advantages of progressive image compression? Demonstrate, with the aid of an example using the sample image below, how a simple hierarchical (pyramid) coding works on both encoding and decoding ends. Assume a 3-layer pyramid structure and use the first elements as representatives. Show all your work. [15]

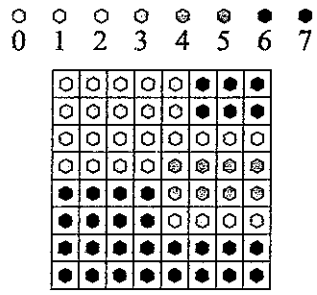
0	0	0	0	2	4	7	7
0	0	0	0	2	5	6	7
0	0	1	1	2	2	5	7
0	0	1	1	2	2	6	7
0	0	1	1	2	2	7	7
0	0	3	3	2	4	6	7
0	0	4	2	2	3	7	7
0	0	5	1	2	2	7	7

Question 3

(a) Derive step by step a canonical minimum-variance Huffman code for the message ABAABCCDEF. Is the Huffman code optimal for the source? If yes, explain why the Huffman code is optimal. Otherwise, describe a source for which the Huffman code would be optimal. Justify your answer. [10]

(b) Describe the main ideas of the *predictive encoding* and *decoding*.

As an example, demonstrate how the image below can be compressed using the predictive coding. Explain how a predictive encoding may improve the compression ratio in terms of entropy change. Show all your work. [15]



Question 4

Consider the two 8×8 matrices (a) and (b) below that represent two image sources respectively. The matrix entries represent the pixel colour values of the images.

(a)

0	0	0	0	2	4	7	7
0	0	0	0	2	5	6	7
0	0	1	1	2	2	5	7
0	0	1	1	2	2	6	7
0	0	1	1	2	2	7	7
0	0	3	3	2	4	6	7
0	0	4	2	2	3	7	7
0	0	5	1	2	2	7	7

(b)

0	0	0	0	2	4	7	7
0	0	0	0	2	5	6	7
0	1	1	1	2	2	5	7
1	1	1	1	3	4	6	7
1	2	1	1	4	5	7	6
2	2	3	3	4	4	6	6
3	4	4	2	3	4	7	6
5	5	5	6	5	7	6	5

Show all your work in answering the following questions.

(a) What is the minimum number of bits required for fixed-length coding for each of the two sources? [2]

(b) Draw a histogram (or bar-chart) to contrast the probability distribution of the two sources, plotting probability values against pixel colour values. [4]

(c) Which source would contain more information on average? Justify your answer. [4]

(d) Compress the two sources applying the canonical minimum-variance Huffman encoding algorithm directly. On which source does the Huffman algorithm achieve a better compression result? On which source does the Huffman algorithm achieve a performance that is closer to optimal? Justify your answers. [15]

Question 5

(a) Outline the Arithmetic decoding algorithm for a binary source in a flowchart.

A binary sequence of length 4 (symbols) was encoded on the binary alphabet (B, W) using the Arithmetic encoding algorithm. Suppose that the probability $Pr(B)$ is computed based on a previous input sequence BWWWWBWWWW and the encoded output is 0.34.

Demonstrate, with the aid of a diagram or a table, how the Arithmetic decoding algorithm derives the original sequence of symbols step by step. [10]

(b) Explain why the Reflected Gray code is a good representation for coding the colours of greyscale images. Derive the *Reflected Gray code* for the decimal number 11. [5]

(c) Explain briefly what is meant by a *cartoon-like image*. Give an example of such an image in real life. Provide a 5×5 matrix of data to aid your explanation. [5]

(d) Explain, with an example of data or diagram, the concept of *data rate* in the context of audio compression. What does the data rate measure? [5]

2910325 [Eastern] 2009 **END OF EXAMINATION**