# Chapter 1

# Introduction

Contents



In this very brief introduction, I aim to give you an idea of the nature of this subject and to advise on how best to approach it. I also give general information about the contents and use of this subject guide, and on recommended reading and how to use the textbooks.

## 1.1 This subject

### Relationship to previous mathematics courses

If you are taking this unit as part of a BSc Degree you will already have taken the pre-requisite Mathematics subject **116 Abstract Mathematics**. In **Abstract Mathematics** you will have learned about the fundamentals of mathematical analysis: in particular, you will have studied sequences and functions, limits and continuity. This subject continues study in mathematical analysis, extending the material in **Abstract Mathematics**. The emphasis is on series, functions and sequences in $n$-dimensional real space, and we shall also study the general (and unifying) concept of a metric space.

After studying this subject, you should be equipped with a knowledge of concepts (such as continuity and compactness) which are central not only to further mathematical courses, but to applications of mathematics in economics and other areas. For example, as we shall see, compactness is a very important idea in optimisation. The subject will also enable you to set the real analysis you previously encountered in a larger context, to see that there is a 'bigger picture'. More generally, a course of this nature, with the emphasis on abstract reasoning and proof, will help you to think in an analytical way, and be able to formulate mathematical arguments in a precise, logical manner.

### Aims

The unit is designed to enable you to:

- develop further your ability to think in a critical manner;
- formulate and develop mathematical arguments in a logical manner;
- improve your skill in acquiring new understanding and expertise;
- acquire an understanding of advanced mathematical analysis.

## Learning objectives

Having taken this subject, you should:

- have a good knowledge of mathematical concepts in real analysis;
- be able to use formal notation correctly and in connection with precise statements in English;
- be able to solve mathematical problems in real analysis;
- be able to find and formulate simple proofs.

## Topics covered

Descriptions of topics to be covered appear in the relevant chapters.

We study the formal mathematical theory of:

- series of real numbers;
- series and sequences in $n$-dimensional real space $\mathbb{R}^n$;
- limits and continuity of functions mapping between $\mathbb{R}^m$ and $\mathbb{R}^n$;
- the derivative
- topological properties of $\mathbb{R}^n$;
- metric spaces;
- uniform convergence of sequences of functions;

Not all chapters of the guide are the same length. It should not be thought that you should spend the same amount of time on each chapter. I will not try to specify precisely how much relative time should be spent on each: that will vary from person to person and I do not want to be prescriptive. However, as a *very rough* guide (bearing in mind that this must vary from individual to individual), I would suggest that the percentages of time spent on each chapter are something along the lines suggested in Table 1.1.

| Chapter | % |
|---|---|
| 2 | 30 |
| 3 | 10 |
| 4 | 15 |
| 5 | 20 |
| 6 | 15 |
| 7 | 10 |

Table 1.1: Suggested allocation of time on the various chapters

## 1.2 Reading

You are advised to read books. This subject guide is just a guide, and is not a textbook.

There are many books that would be useful for this subject, since Mathematical Analysis is a major component of most university-level mathematics degree programmes. There is no single book that corresponds exactly to this subject as it is treated here, but there are many books that are useful for parts of it.

The following books are recommended.

Bryant, Victor. *Yet Another Introduction to Analysis*. (Cambridge University Press: Cambridge, 1990) [ISBN 052138835X]

Binmore, K.G. *Mathematical Analysis: A Straightforward Approach*. (Cambridge University Press: Cambridge, 1982) [ISBN 0521288827]

Brannan, David. *A First Course in Mathematical Analysis*. (Cambridge University Press: Cambridge, 2006) [ISBN 0521684242]

Bartle, R.G. and D.R. Sherbert. *Introduction to Real Analysis*. (John Wiley and Sons: New York, 1999) Third edition. [ISBN 0471321486].

Bryant, Victor. *Metric Spaces: Iteration and Application*. (Cambridge University Press: Cambridge, 1985) [ISBN 0521318971]

Sutherland, W. A. *Introduction to Metric and Topological Spaces*. (Oxford University Press: Oxford, 1995) [ISBN 0198531613]

None of these books covers all of the topics in this course. 'Yet Another Introduction to Analysis' was highly recommended for **Abstract Mathematics**, so you may already have it. It will be useful for Chapters 2 and 4, and it will also be useful for revising the material you will need to know from **Abstract Mathematics**. The Binmore book will be useful for Chapters 2, 3 and 4. Brannan's book will be useful for Chapters 2 and 4. The book by Bartle and Sherbert will be useful for Chapters 2, 4 and 7, and will also be of some use for Chapters 5 and 6. Of more use for Chapters 5 and 6 are the 'Metric Spaces' book of Bryant and the Sutherland book. Note, however, that most of the Sutherland book covers more advanced topics than this subject, and the Bryant Metric Spaces book takes a slightly different approach from that taken here.

## 1.3 Using the guide

The sample questions at the end of the chapters of this guide are a very useful resource. You should try them once you think you have mastered a particular chapter. Really try them: don't just simply read the solutions provided. Make a serious attempt before consulting the solutions. Note that the solutions are often just sketch solutions, to indicate to you how to answer the questions. However, in the examination, you must show all your reasoning. It is vital that you develop and enhance your problem-solving skills and the only way to do this is to try lots of examples.

At certain points I have tried hard to emphasise certain pitfalls that students can fall into in their understanding of key concepts and I have indicated these by the use of the word 'WARNING!'. Please pay particular attention to these: the concepts and results in this subject can sometimes be quite subtle and misunderstandings can easily arise.

You will need to know, and be able to reproduce, some of the proofs of standard results, as indicated in the learning outcomes. I have also included in this guide some other proofs that you will not be expected to reproduce. I have done so in order to help you understand the material better.

## 1.4 Examination

**Important:** the information and advice given in the following section is based on the examination structure used at the time this guide was written. Please note that subject guides may be used for several years. Because of this we strongly advise you to check both the current Regulations for relevant information about the examination, and the current Examiners' reports where you should be advised of any forthcoming changes. You should also carefully check the rubric/instructions on the paper you actually sit and follow those instructions.

Two sample examination papers are given as an appendix to this guide. There are no optional topics in this subject: you should study them all. The examination paper will provide some element of choice as to which questions you attempt: see the Sample Examination Papers at the end of the Guide for an indication of the structure of the examination paper.

Please do not assume that the questions in a real examination will necessarily be very similar to these sample questions. An examination is designed (by definition) to test you. You *will* get examination questions unlike questions in this guide and each year there *will* be examination questions different from those in previous years. The whole point of examining is to see whether you can apply knowledge in familiar *and* unfamiliar settings. For this reason, it is important that you try as many examples as possible, from the guide and from the textbooks. This is not so that you can cover any possible type of question the examiners can think of! It's so that you get used to confronting unfamiliar questions, grappling with them, and finally coming up with the solution.

Do not panic if you cannot completely solve an examination question. There are many marks to be awarded for using the correct approach or method.

## 1.5 The use of calculators

You will not be permitted to use calculators of any type in the examination. This is not something that you should panic about: the examiners are interested in assessing that you understand the key concepts, ideas, methods and techniques, and will set questions which do not require the use of a calculator.