Chapter 1

Introduction

Contents

In this very brief introduction, I aim to give you an idea of the nature of this subject and to advise on how best to approach it. I give general information about the contents and use of this subject guide, and on recommended reading and how to use the textbooks.

1.1 This subject

Relationship to previous mathematics courses

You will have already taken a pre-requisite Mathematics subject, probably the combination of **05A Mathematics 1** and **05B Mathematics 2** (or the **D21 Mathematics** paper on the Diploma in Economics, or the paper **12 Mathematics for Economists**).[1] The title 'Further mathematics for economists' might suggest to you that this subject is more of the same. This is true to some extent. However, for this subject you will not simply have to solve problems: you will have to *prove* things, and that is a significant conceptual step up from the previous mathematics courses you will have taken. This will require you to take a more formal approach to mathematics, and you will note that throughout this Guide I have been rather more formal than in the subject guides for the pre-requisite subjects. In this subject, we need to work with *precise definitions*, and you will need to know these. Not only will you need to know these, but you will have to understand them, and be able (through the use of them) to demonstrate that you understand them. Simply learning the definitions without understanding what they mean is not going to be adequate. I hope that these words of warning don't discourage you, but I think it's important to make it clear that this is a subject at a higher level than those prerequisite subjects.

[1] I will generally assume, as will be the case for most of you, that the pre-requisite mathematics papers you have taken are **05A Mathematics 1** and **05B Mathematics 2**. But the other papers I have mentioned (**12 Mathematics for Economists** and **D21 Mathematics** on the Diploma in Economics) are broadly the same. Note that **74 Quantitative Methods** is not a valid prerequisite. I shall also use the words 'paper', 'subject' and 'course' interchangeably. The 'official' term is 'subject', but I find that rather awkward to use sometimes.

Aims

This subject is intended to:

- enable students to acquire skills in further methods of calculus and linear algebra, as required for their use in advanced economics-based subjects
- enable students to understand the underlying theory behind these

techniques and those of more basic mathematics courses (such as **05A Mathematics 1** and **05B Mathematics 2**)

- prepare students for advanced study in theoretical aspects of economics-based subjects.

Learning objectives

Having taken this subject, students should have:

- knowledge of the subject matter, terminology, techniques and conventions covered in the subject
- the ability to solve problems involving understanding of the concepts and application of the methods
- the ability to prove statements and to formulate precise mathematical arguments.

Topics covered

Descriptions of topics to be covered appear in the relevant chapters. However, it is useful to give a brief overview at this stage.

There are three main components to this subject:

- *Linear algebra*, Chapters 2 to 7.
- *Functions and mathematical analysis*, Chapters 8 and 9.
- *Optimisation*, Chapters 10 and 11.

Throughout, the emphasis is on the theory behind methods that find applications in economics, finance, and other fields. That is to say, the aim in this subject is not only to provide you with techniques and methods which find application in economics and related areas, but to enable you to understand why, and under what circumstances, these techniques (and those you encountered in your earlier mathematical study) work.

The *Linear algebra* chapters place much emphasis on the theory of vector spaces, the theory of matrix diagonalisation, and the applications of matrix diagonalisation. You will certainly have met matrices and linear equations before, but this subject takes these topics a lot further.

The *Functions and mathematical analysis* chapters will be new to you. Chapter 8, on mathematical analysis, is particularly demanding. In this chapter, the aim is to develop a deeper understanding of the techniques used in previous mathematics courses, through the formal study of limits, continuity, and compactness. Chapter 9 concerns calculus of several variables, with consideration given to geometrical interpretations of the concepts.

The *Optimisation* chapters concern optimisation of functions of several variables, and the theory of linear programming. In previous mathematics courses, you studied unconstrained optimisation of two-variable functions and the Lagrange method for constrained optimisation. These topics are taken much further in this subject, and related new topics such as the 'envelope theorems' are discussed. The linear programming chapter is concerned with the theory behind linear programming. Of particular interest is the duality theorem. Applications are given to matrix games.

You will notice that some chapters are significantly larger than others. It should not be thought that you should spend the same amount of time on each chapter. I will not try to specify how much relative time should be spent on each: that will vary from person to person and I do not want to be prescriptive.

As a *very rough* guide (bearing in mind that this must vary from individual to individual), I would suggest that the percentages of time spent on each chapter are something along the lines suggested in Table 1.1. (This should not be taken as any indication about the composition of the examination.)

Chapter	%
2	5
3	10
4	5
5	10
6	10
7	5
8	25
9	10
10	10
11	10

Table 1.1: Suggested allocation of time on the various chapters

1.2 Reading

There are many books that would be useful for this subject, and the subject does not follow any particular one (though particular chapters of this guide follow closely the development of a topic as it is presented in a text). Most topics in this subject are covered in great detail by a great number of books. For that reason, I have resisted the temptation to specify *essential* reading in each chapter of the guide. What is true, however, is that *textbook reading is essential*. Textbooks will provide more in-depth explanations than you will find in this guide (which is explicitly not meant to be a textbook), and they will also provide many more examples to study and exercises to work through. The following books are the ones I have referred to in this guide (listed roughly in order of usefulness).[2]

[2]Earlier editions than those listed here are equally useful.

Simon, C.P. and L. Blume, L. *Mathematics for Economists*. (W.W. Norton and Company: New York and London, 1994) [ISBN

0393957330].

Anton, H., *Elementary Linear Algebra*. (John Wiley and Sons: New York, 2000) Eighth edition. [ISBN 0471170550].[3]

[3] There are many editions and variants of this book, such as the 'Applications version'. All are useful, but you will not need more than one of them. You can find the relevant sections in a different edition by using the index.

Ostaszewski, A. *Advanced Mathematical Methods*. (Cambridge University Press: Cambridge, UK, 1991) [ISBN 0521247888 (hardback)] and [0521289645 (paperback)].

Bartle, R.G. and D.R. Sherbert. *Introduction to Real Analysis*. (John Wiley and Sons: New York, 1999) Third edition. [ISBN 0471321486].

Bryant, V., *Yet Another Introduction to Analysis*. (Cambridge University Press: Cambridge, UK, 1990) [ISBN 0521381665 (hardback)] and [052138835X (paperback)].

Ostaszewski, A. *Mathematics in Economics: Models and Methods*. (Blackwell: Oxford, 1993) [ISBN 0631180559 (hardback)] and [0631180567 (paperback)].

Rowcroft., J.E., *Mathematical Economics: an Integrated Approach*. (Prentice Hall: New Jersey, 1994) [ISBN 0130639672].

Anthony, M. and Biggs, N. *Mathematics for Economics and Finance: Methods and Modelling*. (Cambridge University Press: Cambridge, UK, 1996) [ISBN 0521551137 (hardback)] and [0521559138 (paperback)].

I would recommend that you obtain a copy of Simon and Blume, and also Anton. You will also find it useful to have access to a text on mathematical analysis. The books by Bryant and Bartle and Sherbert are good, but there are many other good books on this topic. The Anthony and Biggs book is useful for revision (being the key text for the prerequisite mathematics papers).

I have already mentioned that it is important to be able to prove things in this subject. Some details of proof techniques can be found in Appendix A1, Section A1.3 of Simon and Blume (but you should not concern yourself with proof by induction). Bartle and Sherbert also have a useful appendix on 'Logic and proofs', should you want a more formal treatment of proof.

1.3 Using the guide

I have already mentioned that this guide is not a textbook. It is important that you read textbooks in conjunction with the guide and that you try problems from the textbooks. The sample questions at the end of the chapters of this guide are a very useful resource. You should try them once you think you have mastered a particular chapter. Really try them: don't just simply read the solutions provided. Make a serious attempt before consulting the solutions. Note that the solutions are often just sketch solutions, to indicate to you how to answer the questions, but in the examination, you must show all your calculations.

It is vital that you develop and enhance your problem-solving skills and the only way to do this is to try lots of examples.

1.4 Examination

Important: the information and advice given in the following section are based on the examination structure used at the time this guide was written. However, the University can alter the format, style or requirements of an examination paper without notice. Because of this we strongly advise you to check the rubric/instructions on the paper you actually sit.

A sample examination paper is given as an appendix to this guide. Many of the questions in this sample paper are given in individual chapters, and for these you will find sketch solutions in the relevant chapters. There are no optional topics in this subject: you should do them all. The examination format is designed with this in mind. There is a compulsory section covering the broad range of the subject, with no choice of questions, and there is an optional section with larger questions and some element of choice.

Please do not imagine for a moment that the questions in a real examination will necessarily be very similar to these sample questions. An examination is designed (by definition) to test you. You *will* get examination questions unlike questions in this guide. The whole point of examining is to see whether you can apply knowledge in familiar *and* unfamiliar settings. The examiners (nice people though they are) have an obligation to surprise you! For this reason, it is important that you try as many examples as possible, from the guide and from the textbooks. This is not so that you can cover any possible type of question the examiners can think of! It's so that you get used to confronting unfamiliar questions, grappling with them, and finally coming up with the solution.

Do not panic if you cannot completely solve an examination question. There are many marks to be awarded for using the correct approach or method.

This is a challenging subject, but once you have mastered it, you will have a new insight into the mathematical techniques that are used in economics and finance and, although this might sound ambitious, you will hopefully be able to think in an analytical and abstract way about problems, a useful skill whatever your future career.

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