## Contents

**Introduction** ............................................................................................................ 1  
  Aims and objectives .................................................................................................. 2  
  Learning outcomes .................................................................................................. 2  
  About levels of knowledge ..................................................................................... 2  
  Methods of writing .................................................................................................. 3  
  About economics ..................................................................................................... 4  
  Structure of the guide ............................................................................................. 4  
  Reading .................................................................................................................... 5  
  Online study resources ........................................................................................... 6  
  Working with others ................................................................................................ 7  
  Examination advice ................................................................................................. 7  
  Some basic mathematical tools ............................................................................... 8  

**Technical preface** ................................................................................................. 9  
  Learning outcomes ................................................................................................. 9  
  Introduction ............................................................................................................. 9  
  Sets and specifications ............................................................................................ 9  
  Numbers .................................................................................................................. 11  
  A point in a plane .................................................................................................... 13  
  Functions and graphs ............................................................................................. 14  
  Self-assessment ...................................................................................................... 21  

**Chapter 1: The study of economics** ................................................................... 23  
  Learning outcomes ................................................................................................. 23  
  Reading ..................................................................................................................... 23  
  Economics as a theory ............................................................................................ 23  
  The fundamental economic problem ..................................................................... 28  
  Specialisation and trade ......................................................................................... 36  
  The shape of the PPF and the importance of marginal changes ......................... 39  
  Self-assessment ...................................................................................................... 42  
  Test your understanding ....................................................................................... 42  
  Answers ................................................................................................................... 44  

**Chapter 2: Individual choice** ............................................................................ 49  
  Learning outcomes ................................................................................................. 49  
  Reading ..................................................................................................................... 49  
  The role of demand ................................................................................................. 49  
  Rationality ............................................................................................................... 53  
  Preferences: the relationship individuals have with the world of economic goods .... 56  
  Deriving demand for economic goods .................................................................. 68  
  Market demand ...................................................................................................... 78  
  Self-assessment ...................................................................................................... 82  
  Answers ................................................................................................................... 84  

**Chapter 3: Production and the behaviour of the firm** ..................................... 93  
  Learning outcomes ................................................................................................. 93  
  Reading ..................................................................................................................... 93  
  Production functions ............................................................................................. 93
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Market structures</td>
<td>125</td>
</tr>
<tr>
<td>5</td>
<td>The market for factors</td>
<td>169</td>
</tr>
<tr>
<td>6</td>
<td>General equilibrium and welfare economics</td>
<td>191</td>
</tr>
<tr>
<td>7</td>
<td>Externalities and public goods</td>
<td>223</td>
</tr>
<tr>
<td>8</td>
<td>Aggregation and the macroeconomic problem</td>
<td>241</td>
</tr>
<tr>
<td>9</td>
<td>The determinants of output</td>
<td>251</td>
</tr>
</tbody>
</table>
Appendix: Sample examination paper ............................................................... 385
Section A.............................................................................................................. 385
Section B.............................................................................................................. 387
You are about to embark on the study of *Introduction to economics*. Economics is a discipline which deals with the broad issue of resources allocation. Within it, an ongoing debate is raging over the question of how best to organise economic activities such that the allocation of resources will achieve that which society desires. A debate which feeds into political discussions in a way that exposes all members of society to the consequences of economic analysis. The academic side of Economics provides the concepts, tools of analysis and reasoning upon which such a debate is based. To be able to understand the logic of an existing system or the motivation behind the drive for its change one must possess a reasonable understanding of economics as an academic discipline. Beside the obvious benefits to society from having better informed citizens, such an understanding can provide one with the ability to benefit most from the system; an ability and drive which are naturally taken into account in economic analysis.

To some of you, economics is not the main area of study and this introductory course is just one of those things which you have to endure in order to receive the academic qualification. May I remind you that the purpose of an academic programme is not to tell you what various things are. Instead, its aim is to help you develop academic skills, the most important of which is a creative and critical way of thinking about almost anything. The fact that not all students are therefore required to take courses only in mathematics, logic and philosophy is merely an indication that nowadays, we have a more sophisticated conception of what critical and creative thinking means. We came to realise that different areas of our interest have their own particular features which are necessary for the development of relevant academic skills. Of course, studying mathematics, logic and philosophy will not reduce one’s critical abilities but they cannot provide the entire scope of considerations which the social sciences demand. Learning what things are will provide you with some knowledge but will not provide you with the skill of analytical thinking. Therefore, the academic programme has been carefully design to provide students of the social sciences with the necessary exposure to the more fundamental methods of analysis that will, we hope, equip you for life with an ability to understand the broad dimensions of society, contribute to it and benefit from it. The implications of this is that the course which you are now beginning to study will sometimes appear intimidating. It is indeed a complex subject. Still, it is our view (and experience) that with patience and work everyone can gain the necessary command over it.

The purpose of this **subject guide** is to assist you in your endeavour and to guide you through the labyrinth of material, levels of knowledge and examination standards. There are, as I am sure you know, numerous textbooks at the introductory level. However, most of them cater for the American market with its unique characteristics and in particular, the notion of general undergraduate studies. This is in contrast with the British (and European) system where degrees are specialised. This means that the level of knowledge, in economics, which is required of a student by the end of their study is much greater than that which would be required of them had they pursued a general degree. Consequently, the
spacing of that knowledge over three years requires a much more rigorous introductory course than is offered by most textbooks. I would therefore strongly advise against picking a single textbook and concentrating one’s effort on it. Instead, you should conduct your study along the lines and recommendations of this subject guide. In it you will find a well-focused organisation of the subject which will highlight those things which we deem to be important. You will find, on each topic, references to readings from a set of textbooks which will help you understand each topic through the use of different methods of exposition. At the end of each topic you will find worked-out past exam questions which will enable you to enhance your understandings as well as help you prepare yourself for the examination.

There are a few sections in the subject guide which are slightly more difficult than others. They are there because we wish to cater for the interested student as much as we would like to support the one who is struggling. We believe that as time is an important factor in the learning process, even the struggling student will reach the point in time where they will wish to expand their knowledge. Naturally, as we must distinguish between the process and learning from the process of assessment, the sections in the guide which we deem difficult will be clearly marked. If they are not essential for examination purposes, you will be advised that you may skip the section and come back to it at your leisure.

Aims and objectives

The aims of this course are to:

• introduce students to an understanding of the domain of economics as a social theory
• introduce students to the main analytical tools which are used in economic analysis
• introduce students to the main conclusions derived from economic analysis and to develop students’ understanding of their organisational and policy implications
• enable students to participate in debates on economic matters.

Learning outcomes

At the end of this course and having completed the Essential reading and activities, you should be able to:

• define the main concepts and describe the models and methods used in economic analysis
• formulate problems described in everyday language in the language of economic modelling
• apply and use the main economic models used in economic analysis to solve these problems
• assess the potential and limitations of the models and methods used in economic analysis.

About levels of knowledge

In contrast with the breadth of some introductory textbooks, Introduction to economics is much more focused. This means that instead of getting acquainted with a little bit about a lot of things, we
wish you to gain real command over fewer things. The key difference here is between ‘getting acquainted’ and ‘gaining command’. For the former, one would normally need to know about economic concepts. To ‘gain command’, however, we want students to know the concepts. Evidently, there is a profound difference between studying for these two kind of purposes.

To know about economics it is indeed sufficient to read about the various economic concepts. Then, whenever you encounter them you will understand what is meant by these concepts. Almost like being able to recognise the meaning of words in a foreign language. But this, as I am sure you will agree, is far from being sufficient in order to be able to speak the foreign language. To achieve this, one would have to learn a bit of grammar too. Most textbooks tend to teach the ‘words’ which are used in economics. We wish to teach you its ‘grammar’.

To know what the concepts are one must not only acquaint one’s self with the meaning of these concepts but one must also be able to use them. This means that after learning about the concept, one must do as many exercises as possible. Exercises, however, can sometimes be misleading. A question like ‘explain the meaning of concept A’ is not an exercise question. An exercise is a problem where the student is expected to:

a. choose the right model, or concept, with which to deal with the problem
b. use the model, or the concept, to derive a solution to the problem.

In this subject guide, you will find such exercises. You will also be provided with the answer. However, to make full use of the guide it is recommended that before you examine our solution to the problem, you try to solve it yourself. When you then compare your own solution to the one which we propose, if they do not match, it is not sufficient for you to say ‘Oh, now I understand the answer’. You probably have only obtained what we may call passive understanding. To reach the level of active understanding you must go over your own solution and try to understand what it is that led you to answer the way you did. Only by clearing away embedded misconceptions will the road be clear to learn the new language.

Methods of writing

The essay-type or discursive writing is a method of exposition becoming the ‘getting acquainted’ approach. In such a format, one tends to write about things and to describe them. For the other approach – which requires active understanding – one would need to resort to a more analytical form of discourse. A form of discourse where the student is ‘making a point’ or, to use a more traditional word from rhetoric, where one is trying to persuade.

To think about writing in this way will help a great deal. It forces the student first to establish what it is that they wish to say. Once this has been established, the writer must find a way of arguing the point. To ‘make a point’, as one may put it, basically means to know the answer to the questions before one starts writing. It is my impression from past examination papers that many students try to answer questions while they are writing the answer. Any question normally triggers a memory of something which one had read in the textbook. It somehow opens the floodgates and students tend to write everything they know about the subject with little reference to what the question is really about. This is not what this course is all about. We want the student to identify the tools
of analysis which are relevant in each question; we want them to show us that they know what these tools are; and, lastly, we want students to be able to use the tools.

The examination questions are normally designed in such a way that will allow the Examiners to view those different levels of student’s understanding. Questions are written in a ‘problem’ form which then require that the student will be able to establish which framework analysis is more appropriate to deal with which problem. In their exposition, students are then expected to properly present this framework. Only then are they expected to ‘solve the problem’ within this framework. Although some questions may have a general appeal, we do not seek general answers. You must think of the examination as an exercise rather than a survey.

About economics

Economics is a broad subject. A quick glance at some of the major textbooks is sufficient to make even the bravest of students faint. Apart from the scary geometrical and algebraic expositions, there is the issue of quantity. The subject matter of economics appears to be so enormous that one begins to wonder whether studying it is not just another form of Sisyphean work.1

While it is true that the subject matter of economics is so broad it does not follow that the study of it should become so laborious. What exactly is economics? The answer is that economics is basically a way of thinking. In the narrow sense of the word it is a way of thinking about those things which are defined as economics activities. In a broader sense, it is a method of thinking about all questions concerning the organisation of society. The scope of the subject, therefore, may sometimes appear as almost unlimited. However, the subject itself – the principles of analysis – is very well defined and well under control.

The purpose of this course is to introduce the student to the fundamentals of economics analysis. This means that what we are concerned with is the study of the way economics think rather than the extent of what they have said.

This subject guide will help you in this endeavour as I intend to highlight the analytical points while spending less time on applying those principles to various social issues. It is a kind of alternative textbook. The precondition for passing the final exam is to have a good command of all things which are presented in this subject guide.

Structure of the guide

With the exception of Chapters 1 and 6, you should start by doing the suggested reading given at the beginning of each chapter. Then, you should work through the relevant chapter in the subject guide, attempting the questions and exercises throughout.

Each chapter begins by listing the main points of the issue being discussed. You should always go back to this list after you have worked through the chapter to ensure that you have a satisfactory understanding of each of these items before you move to the next chapter.

You will, no doubt, find some parts of this subject guide slightly more difficult than others. This is because we wish to cater for students at all levels of ability. Naturally, as we must distinguish the process of learning from the process of assessment, the sections in the guide which
we consider difficult will be clearly marked. If they are not essential for examination purposes, you will be advised that you may skip the section and come back to it at your leisure.

When you have a good grasp of the discussed subjects, and the corresponding readings, you should explore the textbooks in more depth.

**Reading**

Some of the larger economics textbooks reflect a mixed view of what an introductory course in economics should look like. While providing the fundamentals of economic analysis, they also try to show the scope of the subject. This means that a lot of the material in these textbooks is not really part of this subject, and rather serves to illuminate some of the ways economic analysis can be used to look at society.

So the good news is that a great deal of what appears in some of the books is of less interest to us. The not-so-good news is that as a language and a method of analysis, logic (and hence mathematics) is an important component of our subject. Still, most of the logical arguments can also be presented in a less formal way. Therefore, although mathematics lies at the heart of the subject, mathematical expositions are not an essential part of learning the language of economics.

In short, the heart of the subject guide is the study of economic reasoning. This means that the extent of the subject guide is much reduced, compared to some of the more comprehensive textbooks. On the other hand, this subject guide is more rigorous than some of the textbooks. You should always carefully check your understanding of each step of the analysis, you should never accept a proposition without understanding the logic behind it.

**Recommended reading**

You are strongly advised to stick to **one** of the two textbooks listed below for your additional reading. Only look at the other textbook if you find a topic difficult and feel that the teaching style in the other book suits you better. It is not important to read a huge amount beyond the subject guide, but it is very important to really understand what you do read.


Detailed reading references in this subject guide refer to the editions of the set textbooks listed above. New editions of one or more of these textbooks may have been published by the time you study this course. You can use a more recent edition of any of the books; use the detailed chapter and section headings and the index to identify relevant readings. Also check the virtual learning environment (VLE) regularly for updated guidance on readings.

Please note that there is a textbook which is based on the subject guide but which goes well beyond it. It brings together the learning of the tools and their practice through solved self-assessment exercises. The book's details are:

Online study resources

In addition to the subject guide and the Essential reading, it is crucial that you take advantage of the study resources that are available online for this course, including the VLE and the Online Library.

You can access the VLE, the Online Library and your University of London email account via the Student Portal at:

http://my.londoninternational.ac.uk

You should receive your login details in your study pack. If you have not, or you have forgotten your login details, please email uolia.support@london.ac.uk quoting your student number.

The VLE

The VLE, which complements this subject guide, has been designed to enhance your learning experience, providing additional support and a sense of community. It forms an important part of your study experience with the University of London and you should access it regularly.

The VLE provides a range of resources for EMFSS courses:

- Self-testing activities: Doing these allows you to test your own understanding of subject material.
- Electronic study materials: The printed materials that you receive from the University of London are available to download, including updated reading lists and references.
- Past examination papers and Examiners' commentaries: These provide advice on how each examination question might best be answered.
- A student discussion forum: This is an open space for you to discuss interests and experiences, seek support from your peers, work collaboratively to solve problems and discuss subject material.
- Videos: There are recorded academic introductions to the subject, interviews and debates and, for some courses, audio-visual tutorials and conclusions.
- Recorded lectures: For some courses, where appropriate, the sessions from previous years' Study Weekends have been recorded and made available.
- Study skills: Expert advice on preparing for examinations and developing your digital literacy skills.
- Feedback forms.

Some of these resources are available for certain courses only, but we are expanding our provision all the time and you should check the VLE regularly for updates.

Making use of the Online Library

The Online Library contains a huge array of journal articles and other resources to help you read widely and extensively.

To access the majority of resources via the Online Library you will either need to use your University of London Student Portal login details, or you will be required to register and use an Athens login:

http://tinyurl.com/ollathens

The easiest way to locate relevant content and journal articles in the Online Library is to use the Summon search engine.
If you are having trouble finding an article listed in a reading list, try removing any punctuation from the title, such as single quotation marks, question marks and colons.

For further advice, please see the online help pages: www.external.shl.lon.ac.uk/summon/about.php

**Working with others**

Group work is an important element of effective learning. Of course, you can study the material on your own, but discussing problems and insights with others is important for two reasons. First, it exposes you to different ways of thinking about the same problem. Second, it forces you to convince others about your own line of argument. The process of trying to convince others will enable you to gain a much deeper understanding of the material you are studying.

Even if there are not enough people around you who study the same subject, it would still be useful if you could persuade at least one other person to work with you. Try to explain to the other person what you have been learning. If you succeed in teaching them economics, you will have done very well.

**Examination advice**

**Important**: the information and advice given here are based on the examination structure used at the time this guide was written. Please note that subject guides may be used for several years. Because of this we strongly advise you to always check both the current *Regulations* for relevant information about the examination, and the VLE where you should be advised of any forthcoming changes. You should also carefully check the rubric/instructions on the paper you actually sit and follow those instructions.

Remember, it is important to check the VLE for:

- up-to-date information on examination and assessment arrangements for this course
- where available, past examination papers and *Examiners’ commentaries* for the course which give advice on how each question might best be answered.

Many subjects, and their exams, require essay-type answers, in which one tends to write **about** things and to describe them. For this course, the approach is different, and you need to adopt a more analytical form of discourse, which aims to persuade and to ‘make a point’.

Thinking about writing in this way will help you a great deal. It forces you to think about what you want to say, as well as about how you will argue your point. ‘Making a point’ requires you to basically know the answer **before** you start writing. It is my impression, from past examination papers, that many students try to answer questions **while** they are writing the answer. Reading a question normally triggers memories of things which you have read in the textbook. This often leads students simply to write everything down that is remotely connected to the question, with little reference to the problem the question actually poses.

This is **not** what this course subject is all about. We want you to:

- identify the tools of analysis which are relevant to each question
- show us that you know what these tools are
- be able to use the tools.
The exam questions are normally designed to allow the Examiners to see those different levels of understanding. Questions are written in a ‘problem’ form, which requires you to be able to establish which framework of analysis is most appropriate. In your answer, you are expected to properly present this framework. Only then are you expected to ‘solve the problem’ within this framework. Although some questions may have a general appeal, we do not seek general answers. You must think of the exam as an exercise rather than a survey.

Some basic mathematical tools

Many students find the use of mathematics in economics intimidating. There are no sound reasons for this. Although there is some use of mathematical notation, the level of mathematical analysis which is required is basic. Still, to ensure that technical problems do not create unnecessary obstacles, we recommend that you should focus on clarifying some basic concepts before going any further. These basic concepts include:

- what is a point in a plane
- what is a function, a graph and a slope
- the meaning of a derivative and tangency.

In particular, you must have a good understanding of slopes, as these are the most important tool for understanding the geometrical expositions of the subject. To assist you in reviewing these basic mathematical concepts, the short ‘technical preface’ introduces some of the most basic mathematical and geometrical notions.

We recommend that you begin your study by reading this preface, together with the following references from existing textbooks:

- Lipsey and Chrrystal, (see the Recommended reading) also explains the mathematical tools required to study this subject.

**Do not continue until you are sure that these basic tools are properly understood. When you are sure, continue to the question below.**

**Question 1**

Draw a ‘plane’ figure with y on the vertical axis and x on the horizontal axis. Plot the following points:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>80</td>
<td>4</td>
</tr>
<tr>
<td>60</td>
<td>8</td>
</tr>
<tr>
<td>40</td>
<td>12</td>
</tr>
<tr>
<td>20</td>
<td>16</td>
</tr>
<tr>
<td>0</td>
<td>20</td>
</tr>
</tbody>
</table>

- What is the slope of the line connecting all these points?
- Write the equation which describes this line.
If you are not familiar with the language of economics, work through this short preface before beginning the main part of the guide. Make sure that you thoroughly understand what is being said, and how it is expressed in economics terms. It will be particularly useful in helping you understand the numerous formulas and figures that we use later on.

**Learning outcomes**

At the end of the chapter, you should be able to define and list examples of:

- sets and their enumeration
- natural, integer and rational numbers
- planes and xy-coordinates on a plane
- functions, slopes and binding constraints.

**Introduction**

When we look around ourselves we see many individual things – often more than we can make sense of or communicate clearly to others. We need to find effective ways to think about and describe all these things.

Listen to the scream of a hungry Neanderthal husband to his ‘wife’ in the cave: ‘Dinner, dear!’

For his wife to understand what he wants, they must both know exactly what ‘dinner’ means. She is unlikely to offer him a tree or a stone to eat. She knows, as well as he does, that ‘dinner’ refers to the kinds of things that we eat at a certain time of the day. So instead of the poor wife offering him a random selection of objects from sticks to dung, using the word ‘dinner’ brings the number of objects under consideration down to a manageable number.

‘Dinner’ defines a certain group of objects within the complex world which surrounds us. Of course, this group of objects varies across cultures, but in all of them there will be one word to identify the set of objects from which the meal is likely to be prepared.

Suppose now that ‘dinner’, or ‘things we eat at this time of the day’ includes only two objects: bread and eggs. Would the Neanderthal and his wife consider 100 eggs as ‘dinner’? Probably not. She is more likely to consider ‘two slices of bread and one egg’ as an example of ‘dinner’. So it is not enough just to group those things in the world to which we want to relate. We must also be able to count, or enumerate, them. The two fundamentals here are called **sets** and **numbers**.

**Sets and specifications**

**Sets**

A **set** is a collection of well-defined objects, which are called its **elements** (or **members**).

What is the set, and what are the elements in the dinner example we have just used?
In economics, if $X$ is an element or member of a set $S$, we write:

$$X \in S$$

The negation of this is:

$$X \not\in S$$

which says ‘$X$ is not part of the set $S$’.

In slightly different terms, we could write the broadest definition of our ‘dinner’ set like this:

$$D = \{X | X \text{ is edible}\}.$$  

Put into words, this reads: the ‘dinner set’ contains all ‘things’ which are edible (the vertical line in expressions like these means ‘where’ or ‘such that’ or ‘conditional on’). This therefore says:

Dinner is all $X$ such that $X$ is edible.

But do we eat all things which are edible, or is our taste refined by custom and culture?

**Specifying a set**

We can specify a set in two ways: either by **enumeration** (listing what is in the set) or by **description**.

**Examples of enumeration**

$$A = \{1, 2, 4\}$$

or

$$B = \{\text{Romeo, Juliet}\}.$$  

Here $A$ is the set containing the numbers 1, 2 and 4 and $B$ is the set containing Romeo and Juliet. We have enumerated all the members.

In our dinner example, the set called ‘dinner’ ($D$) may be enumerated like this:

$$D = \{\text{Eggs, Bread}\}.$$  

This does not tell us how many eggs, or bread, constitute a meal. However, the wife not only knows what ‘things’ might constitute the ‘dinner’ set, she also knows her husband’s capacity.

**Description** Suppose that to eat more than 5 eggs in a meal is considered dangerously unhealthy. To eat more than 10 slices of bread might also be inappropriate. The ‘meal’ set – those meals that a good wife will offer her Neanderthal husband – will only contain those meals that are healthy. Hence, the ‘meal’ set, to which both husband and wife are implicitly referring, is a subset of $D$, where $D$ is the set of all possible meals (including unhealthy meals). It is given by the expression:

$$M = \{(E,B) | 0 \leq E \leq 5, 0 \leq B \leq 10\}.$$  

Can you see what the letters $M$, $B$ and $E$ stand for in this expression?

Here $(E,B)$ is a typical member of the meal set comprising Eggs ($E$) and Bread ($B$). Put into words, $M$ is the set of healthy combinations of $E$ and $B$ such that there are between 0 and 5 eggs and between 0 and 10 slices of bread. Clearly the set $M$ is itself contained in, or is a subset of, the set $D$ (we denote this by $M \subseteq D$).
Practise writing down some similar expressions for sets, for example: what will be the set describing the guest list for your dinner party?

Here are some other examples of a **descriptive** way of writing a set, this time a set of solutions to a mathematical problem:

\[ C = \{ X | X^2 - 25 = 0 \} \]

*C* is the set of all the values of *X* that solve the equation \( X^2 - 25 = 0 \). If we add +25 to each side of the equation, the equation becomes: \( X^2 - 25 + 25 = +25 \), which can be reduced to: \( X^2 = 25 \).

The solution of this equation is the square root of 25 (which is either +5 or –5). In this case, we could have **enumerated** the set like this:

\[ C = \{ +5, -5 \} \]

Now consider the set *L*:

\[ L = \{ Y | Y \text{ loves Romeo} \} \]

*L* here is the set of ‘all things that love Romeo’. By enumeration, the set may look like this:

\[ L = \{ \text{Juliet, Romeo, Romeo's mother, Romeo's dog, the girl next door, …} \} \]

For a description of a set to be meaningful, we must have an idea about the **range** of the objects which might be included in the set. In our earlier examples, we must know the possible values of the variables *X* and *Y*:

- In *C*, the **range** of *X* is the set of all **real numbers**.
- For *L*, the **range** of *Y* might be all of the characters in Shakespeare's play *Romeo and Juliet*.
- For our meal set *M*, the range for *E* was all real numbers between 0 and 5 and for bread, all real numbers between 0 and 10.

### Numbers

**Natural numbers**

Numbers are one of the means of describing a set. The most natural way of using numbers is the process of **counting**. The numbers we use for counting (two slices of bread, one egg et cetera) are called **natural numbers**.

The set of **natural numbers** is defined as:

\[ \mathbb{N} = \{ 1, 2, 3, 4, \ldots \} \]

Natural numbers are therefore **positive whole numbers**. But how will you count how much money you have in your bank if you are £200 overdrawn? Well, you are obviously the proud owner of a negative sum: –£200. But while 200 is a natural number, –200 is not: it is whole, but not positive. Perhaps you may dismiss your overdraft as being an unreal number and a capitalist conspiracy.

**Integers**

To allow for circumstances where we want to consider negative numbers, we define a new group of numbers called **integers**. These are all the natural numbers and also their negative values. It also includes the number zero, but we will not discuss this here.

The set of **integers** is defined as:

\[ \mathbb{Z} = \{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \} \]
However, the world around us is too complex to be depicted by integers (whole numbers, whether positive or negative) alone. Rather, the world seems to be continuous. Suppose that the distance between two points (say A and B) is an integer (say 1 mile). Suppose that you live at A and your college is at B. If there is a fast-food outlet halfway, does this mean that you can’t ever have lunch simply because there is no way of describing the distance between your home, or college, and the fast-food outlet? Of course not, there is a distance: it is real, and you can imagine yourself stopping at the fast-food outlet. However, we cannot account for it in a world of integers. We therefore, need to define yet another group of numbers, which can help us to depict the world better. These are called the rational numbers.

**Rational numbers**
The set of **rational numbers** is defined as:

\[ \mathbb{Q} = \{ X/Y | X \in \mathbb{Z} \land Y \in \mathbb{N} \} \]

Put into words, this says that \( \mathbb{Q} \) is the set of fractions \( X/Y \) such that \( X \) is an integer (which can be a negative number) and \( Y \) is a natural number. Thus the set of rational numbers could include any such numbers as: \( 1/2, -1/15, -125/6000 \) or their decimal equivalents.

If the set \( \mathbb{Q} \) contained all possible numbers which we might come across in real life, we could stop here. However, in reality there are also numbers that are not rational – the number \( \pi \) for example. We know that the area of a circle with radius \( r \) is \( A = r^2\pi \), and that when \( r = 1 \), the area of the circle will be \( \pi \). This is real, but cannot be expressed as a rational number. Similarly, \( \sqrt{2} \) and Euler’s Constant \( e \) are not rational numbers.

**Real numbers**
‘Not rational’ means that we cannot obtain the number as a fraction of (or ratio between) integers and natural numbers. All **real numbers** have a decimal expression (for example, \( 12/15 = 0.8 \), and \( 15/11 = 1.36363636… \)). Rational numbers can be defined as real numbers whose decimal expression terminates (as in \( 12/15 \)) or else repeats itself over and over again (as with \( 15/11 \)).

For instance, \( 5/2 \) terminates (it is equal to decimal 2.5) and \( 22/7 \) repeats itself (it is equal to decimal 3.142857 142857 142857). \( \pi \), however, neither terminates nor repeats itself:

\[ \pi = 3.141592653589793… \]

The set of all real numbers, \( \mathbb{R} \), can be represented geometrically, by a straight line, as in **Figure 1**: 

---

\(-\infty \quad +\infty\)

**Figure 1: The real number line.**

We call this line the **real number line**, and it stretches from negative infinity to positive infinity. However, we can also express sets in a geometrical way.
A point in a plane

Sometimes, we define sets of objects across multiple dimensions. For instance, our ‘dinner’ from before contains more than one object. We said that it contains both bread and eggs. If we can count bread and eggs in terms of real numbers then the line which depicts the real numbers will not be sufficient to describe the object called ‘dinner’. We will need two lines: one to count bread, and another to count eggs. The set ‘dinner’ can therefore be written like this, with \( \mathbb{Q} \) standing for ‘rational numbers’:

\[
D = \{ (X, Y) | X \in \mathbb{Q} \land Y \in \mathbb{Q} \}
\]

Write out in words exactly what this expression means.

In words, dinner is a set comprised of \( X \) (the name for bread) which can be counted by real numbers and \( Y \) (the name for eggs), which can be counted by real numbers as well.

‘Dinner’, therefore, is defined by two real number lines, as Figure 2 shows:

![Figure 2: The \((X, Y)\) plane.](image)

The intersecting axes \( X \) (horizontal) and \( Y \) (vertical) are the names of the variables which are enumerated by real numbers. In our ‘dinner’ case, \( X \) stands for ‘slices of bread’ and \( Y \) stands for ‘eggs’. To distinguish between the name of the variable and a particular quantity of it, we use an index number, denoted by a subscript. Hence:

- \( X_0 \) denotes a certain quantity of \( X \)
- \( X_0 \) units of \( X \) may mean ‘10 slices of bread’
- \( X_1 \) will denote another quantity of \( X \), which may or may not be the same as \( X_0 \).

We may add further quantities, called \( X_2, X_3 \) and so on.

But remember that in these expressions, the subscripts 0, 1, 2, 3 and so on do not describe the magnitude of these quantities. They only identify them: it may be better to think of them as the initial, 1st, 2nd quantities respectively.

The two lines of real numbers define what we call a ‘plane’. This plane (of real numbers) is often denoted by \( \mathbb{R}^2 \) (meaning ‘two sets of real numbers’).

A typical point in this plane, say \( A \) in Figure 2, is defined as:

\[ A = (X_0, Y_0) \]

This means that \( A \) is a combination of \( X_0 \) units of \( X \) and \( Y_0 \) units of \( Y \).

Each point in the plane of real numbers has two coordinates. The first one refers to variable \( X \), the second refers to variable \( Y \). This, in turn,
divides the plane into four quadrants. The upper right-hand quadrant contains elements like \( A \) where the coordinates of both variables are positive numbers (including zero, which is both positive and negative at the same time). The bottom right quadrant is where an element in the plane has a positive \( X \) coordinate but a negative \( Y \) coordinate. The third quadrants on the bottom left contains elements for which both variables are assigned a negative number. In the fourth quadrant, \( X \) has negative values while \( Y \) has positive values. In Figure 2, \( X_i \) is a negative number, which is not very meaningful if \( X \) denotes slices of bread.

As far as our ‘dinner’ is concerned, we can rule out any negative consumption. We must, therefore, redefine the ‘dinner’ set to account for positive (including zero) consumption of both bread \( (X) \) and eggs \( (Y) \):

\[
D = \{(X, Y) | (X, Y) \in \mathbb{R}^2_+\}
\]

where \( \mathbb{R}^2_+ \) depicts the positive quadrant in the real numbers plane.

So when our male chauvinist Neanderthal comes to the cave and yells ‘Dinner, dear’, both of them know that he means positive quantities of bread and eggs (the positive quadrant). However, while both of them know what the components of a meal are, the actual composition can vary considerably across cultures and fashions. In other words, what exactly a meal is depends on where, and when, the Neanderthal story takes place.

At this stage, let us consider only the capacity limitations (which are almost universal). To eat more than 10 slices of bread or more than 5 eggs is considered dangerously unhealthy.

The subset called ‘meal’, which is a set contained in the set of all possible ‘dinners’, contains the point \( (0,0) \) but cannot go beyond point \( A \) due to health reasons. Thus a meal cannot include more than 10 slices of bread \( (X \leq 10) \) or 5 eggs \( (Y \leq 5) \). The set \( M \), therefore, is contained in the shaded area of Figure 3, including the edges:

\[
M = \{(X, Y) | 0 \leq X \leq 10, 0 \leq Y \leq 5\}
\]

![Figure 3: The set of possible meals depicted in the (X, Y) plane.](image)

**Functions and graphs**

So far, we have been dealing with how to conceptualise the world around us. We examined some categories through the use of sets and we also used the figures to show that sets can have a geometrical representation. We have not begun, however, to introduce any kind of order to the world. We have not, for example, discussed issues like causality.

‘Causality’ is a very difficult concept, and here we shall only deal with the question of how to represent a causal relationship.
Graphs

Consider the development of a baby. There are many variables which determine its development. How can we tell whether a baby is developing properly? We might think about the two variables of length (height) and weight. A baby may be growing taller, but at the same time not putting on enough weight. Conversely, a baby may be gaining too much weight given that it is not growing in length.

To have a balanced picture, we must observe how well the baby is doing in both important dimensions of its growth. A tool that can help us do so is the graph.

Both length and weight are enumerated by real numbers. Therefore, the development of these two variables will have to be analysed in the real numbers plane, \( \mathbb{R}^2 \). As we know that a negative weight, or length, are meaningless numbers in this context, we can concentrate on the positive quadrant of the real numbers plane, as in Figure 4.

![Figure 4: A depiction of babies' weight-length combinations.](image)

Here \( X \) and \( Y \) denote length and weight of a baby respectively. We have drawn three lines in the plane, to represent the expected growth rates of babies that are relatively large (\( A \)), average (\( B \)), and relatively small (\( C \)) at birth. Each line consists of a set of points in the positive quadrant which have two coordinates each: one for length \( X \) and one for weight \( Y \). The graph lines define sets. In other words, the lines connect a large number (actually an infinite number) of points with different values of \( X, Y \).

From each of the initial points \( A, B \) and \( C \), we may now move along the relevant graph. As we do so, we depict a systematic increase in the values of both variables. This suggests a connection between the weight and length for which we believe the development of a baby is normal given different conditions at birth. (We have omitted the important time dimension, which would have complicated our story. We shall assume that at least in one dimension the baby develops over time.)

The actual progress of any particular baby may not follow any of these lines. We shall have to create a special graph for it. We can then relate the actual development graph to the desired paths and determine how well the baby is developing. This is the thick line labelled 'Actual'.

The graph line, therefore, provides us with a set of points which represent a certain relationship. This does not mean it is a causal relationship. That is to say, it does not mean that the values of \( X \) (length)
determine, or explain, the values of $Y$ (weight) nor that the values of $Y$ explain, or determine, the values of $X$. We simply use the graph to depict the combination of length and weight which constitute our accepted view of balanced growth.

In a similar way we could draw a line within the meal set $M$, which would depict what one may consider a balanced diet (Figure 5).

**Figure 5:** A graph of all balanced diets.

Meals-that-do-not-kill (no more than 5 eggs and 10 slices of bread) are captured by the shaded area in this figure, which represents the set:

$$M = \{(X, Y)| 0 \leq X \leq 10, 0 \leq Y \leq 5\}$$

A balanced diet could mean a balanced consumption of protein (coming from eggs) and carbohydrates (coming from bread). It implies a certain correspondence between the amount of eggs and bread that one eats. The heavy line in this figure depicts such a diet. Again, there is no causal relationship, and the graph simply defines a certain set, the elements of which are comprised of eggs and slices of bread.

**Slopes and functions**

**Slopes**

Consider the subset of balanced diets depicted by the line in Figure 6:

**Figure 6:** Balanced diets again.
Here, the balanced diet is described by the straight line going from the origin, the point \((X = 0, Y = 0)\) or simply \((0, 0)\), to point \(F\) where \(X = 10, Y = 5\) (that is \((10, 5)\)). What can we learn from this line, apart from a detailed list of combinations of bread and eggs which are considered a balanced diet? We can find the value of one thing in terms of its desired relation to another (The desired outcome is a balanced diet).

Notice that according to the line, the following combinations of \(X\) and \(Y\) (among others) constitute a balanced diet:

<table>
<thead>
<tr>
<th>Slices of bread (X)</th>
<th>Eggs (Y)</th>
<th>As a point in the plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>(A = (2, 1))</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>(B = (4, 2))</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>(C = (6, 3))</td>
</tr>
</tbody>
</table>

Suppose that we are consuming 2 slices of bread and 1 egg (point \((2, 1)\)), and we now wish to increase our consumption of bread to 3 slices. Worried about unbalancing our diet with the extra carbohydrate, we would immediately want to compensate for it with some protein (and cholesterol) so that our diet remains balanced. How many more eggs should we consume?

We could easily take a ruler and set it vertically against the point \(X = 3\) and find the corresponding coordinate of \(Y\) which will yield a point on the balanced diet line. This will tell us how many eggs we can consume with 3 slices of bread without breaking our diet. The answer will obviously be to consume 1/2 an egg more (point \(A\)’ in the above diagram).

What if we were consuming 4 slices of bread and 2 eggs (point \((4, 2)\)) and we now want 1/2 a slice of bread more? We could repeat the exercise with the ruler. But even without using the ruler, I can tell you that we would need to consume 1/4 of an egg more.

If you repeat the exercise for any conceivable increase in the consumption of bread from any conceivable point of consumption, you will be able to derive a rule. Doing it in this way means following the logic of induction (from the particular to the general). But we may also be able to establish the rule by deduction (from the general to the particular). What you want to find is how the change in the value (the number) of one variable, say ‘slices of bread’, relates to the change in the value of the other so that we are still in the set of ‘balanced diets’.

Let us consider for a moment the two extremes of the ‘balanced diet line’. At the one end there is point \((0, 0)\) which I shall call point \(O\) and at the other end there is point \(F\) (for Full) where \(F = (10, 5)\). Between \(O\) and \(F\), the value of \(X\) changes by 10 and the value of \(Y\) changes by 5. Hence, \(dX = 10; dY = 5\).

The definition of the slope of a graph is:

\[
\text{slope} = \frac{dY}{dX}
\]

which is, in fact, the tangent of the angle \(\alpha\), \(\tan \alpha\) (see Figure 7). It tells us by how much \(Y\) has changed for a given change of \(X\).
Figure 7: The definition of a slope.

In our case, the slope of the ‘balanced diet line’ (which is the tangent of angle $\alpha$) is:

$$\frac{dy}{dx} = \frac{5}{10} = \frac{1}{2}$$

In maths we may not be interested in the meaning of the number 1/2. In economics, however, we give meaning to mathematics by assigning significance to the various variables. In turn, this assigns meanings to concepts like the slope. If you think carefully, the slope suggests 5 eggs per 10 slices of bread. Or 1/2 an egg per slice of bread, which is the same thing. It gives us some kind of an equivalence scale which is represented by the special line of our ‘balanced diet’. One egg is equivalent, in our ‘balanced diet’, to 2 slices of bread. The operational implications are that if you wish to increase your consumption by 1 slice of bread, you must add 1/2 an egg to your consumption of eggs in order not to deviate from your ‘balanced diet’. If you want 2 eggs, you must add 4 slices of bread. If you want 2.5 eggs, you will have to add 5 slices of bread and so on and so forth.

If you look at points A, B and C in Figure 6, you will find that this general rule (just like the rule of 1/2 an egg per slice of bread) applies everywhere.

The reason for this is simply that the ‘balanced diet’ line is a straight line. The meaning of a straight line is that it has the same slope everywhere. If you now choose any two points along this line you will find that the change ratio of the variables always complies with what we have found: half an egg as an equivalent to one slice of bread.

Functions

Having established a general rule which relates slices of bread $X$ to eggs $Y$, we may want to write the rule in a more explicit fashion. In other words, we want to find a form that will provide a brief, and comprehensive, description of the ‘balanced diet’ line in the above diagram. That is to say, we are searching for a function.

A function is a rule which assigns each element in one set to a unique element in another set. In our case we have two sets. $B$ denotes the set containing various quantities of bread:
B = \{X \mid 0 \leq X \leq 10\}

E denotes the set containing the various quantities of eggs allowed:

E = \{Y \mid 0 \leq Y \leq 5\}

The ‘balanced Diet Function’ \( f \) (stands for function, of course) is a rule which assigns a value in \( E \) to each value in \( B \) (generally denoted by \( f : B \rightarrow E \)). In our case, both \( E \) and \( B \) contain real numbers so \( f \) is a ‘real numbers’ function and we can say that \( f : R \rightarrow R \). So \( f \) is a ‘mapping’ from real numbers to real numbers. It tells us how many eggs we can consume with any possible quantity of bread.

We know that with 0 bread we may consume 0 eggs. But we also know that for every extra slice of bread we must consume an additional 1/2 egg. Hence we write:

\[
Y = f(X) = \frac{1}{2}X
\]

You can now check this function by setting values for \( X \) and finding whether or not the function yields a value of \( Y \) which corresponds to what you would find if you had used a ruler. We can easily see now what role the slope plays in this function. We know that for every change in \( X \) (\( dX \)) we will need a change in the consumption of eggs (\( Y \)) to maintain a balanced diet. We can therefore write:

\[
dY = \frac{1}{2}dX
\]

This means that if we increase the consumption of bread by 1 slice (\( dX = 1 \)), the consumption of eggs (\( Y \)) will have to change by adding 1/2 an egg (\( dY = 1/2 \)).

Divide both sides by \( dX \) and we get:

\[
\frac{dY}{dX} = \frac{1}{2}
\]

which is exactly the slope of the line (the function).

The interpretation which we gave to the slope (as an equivalent scale) is influenced by the nature of the variables as well as by the direction, or sign, of the slope.

Our ‘balanced diet’ concentrated on the balanced intake of carbohydrate and protein. Increased consumption of food (in the dinner set \( M \)) required a simultaneous increase in both variables. It is the fact that the consumption of both bread and eggs had to be increased in order to maintain a balanced diet that forced on us the interpretation whereby 1/2 an egg and 1 slice of bread are equivalent in some way. Equivalence here is an expression of dependency. Whether we stay on our balanced diet when we increase the consumption of one good depends on an equivalent increase in the consumption of the other.

We say that a line has a positive slope whenever the signs of both changes are the same. Here, staying within the boundaries of a balanced diet meant an increase in both \( X \) and \( Y \). As \( dX > 0 \) and \( dY > 0 \),

\[
\frac{dY}{dX} > 0
\]

If instead, we thought of ‘balanced diet’ in terms of calories, the picture would be different. Let \( X \) and \( Y \) represent the same variables (that is, slices of bread and eggs respectively) and suppose that there are 50 kilocalories
in a slice of bread and 80 kilocalories in an egg. Suppose too that a 'healthy diet' means a meal of 400 kilocalories. This is not the same as a balanced diet: this time we are going to set a 'constraint' of a maximum of 400 kilocalories in total. The set $H$, of healthy meals, will be a subset of our original 'dinner' set $D$. Remember that we defined $D$ like this:

$$D = \{(X, Y) \mid X \in \mathbb{Q} \land Y \in \mathbb{Q}\}$$

**Constraints**

The new 'healthy meal' set obviously contains positive amounts of food and is confined to the positive quadrant. However, it now has an additional constraint since the amount of calories derived from the consumption of bread (50 kilocalories per slice times the number of slices, namely, $50X$) and that derived from consuming eggs (80 $Y$), should not exceed 400:

$$H = \{(X, Y) \mid (X, Y) \in D : 50X + 80Y \leq 400\}$$

In words, the set of healthy meals contains all combinations of slices of bread and eggs which are in the dinner set (i.e. the positive values of $X$ and $Y$), provided that the sum of their calories does not exceed 400.

Let us examine first where the constraint is **binding**. We want to find the points where the number of calories allowed has been exhausted. That is, to find the combinations of $X$ and $Y$ for which:

$$50X + 80Y = 400.$$  

We are trying to find a **rule** which will describe the combinations of $X$ and $Y$ for which we consumed the entire quantity of calories which is allowed. The way we have written the constraint automatically reminds us of the idea of a **function**. But this is a very strange function. To turn it into something more familiar, we simply rearrange it:

$$50X + 80Y = 400$$

take $50X$ from both sides

$$80Y = 400 - 50X$$

divide both sides by 80

$$Y = f(X) = 5 - \frac{5}{8}X$$

**Figure 8** describes the function $f$ as well as the set $H$, which is the shaded area:

![Figure 8: A calories constraint.](image-url)
To draw the function, we must know at least two of the following three things: the intercept with the X-axis, the intercept with the Y-axis, and the slope. The intercept with the X-axis denotes the value of X when Y = 0, and the intercept with the Y-axis denotes the value of Y when X = 0. It is easy to establish that if X = 0, Y = 5, i.e. the point (0, 5), and that the slope of this function is \( -(5/8) \). Note that this is a negative number.

Before coming back to the slope let us first draw the line using the two intercepts. We know that (0, 5) is one point on the graph. We can also easily establish the value of X when Y = 0:

\[
0 = Y = 5 - \frac{5}{8}X
\]

\[-\frac{5}{8}X = -5\]

\[X = 8\]

What, then, is the slope of the Healthy Diet Constraint? Since the healthy diet constraint is a straight line, the slope can easily be deduced from the tangent of the angle \( \beta \) in Figure 8, which is clearly \(-5/8\).

Suppose that we increase the consumption of bread by 1 slice (dX = 1). This means that we have added 50 calories to our consumption. To remain on a healthy diet, we must reduce the consumption of eggs (Y). Given that each egg has 80 calories, we will need to reduce this consumption by 5/8 of an egg.

If we change X (dX), we change the value of Y by the coefficient in front of X. In the above equation it is \(-5/8\):

\[
Y = f(X) = 5 - \frac{5}{8}X
\]

\[
dY = -\frac{5}{8}dX
\]

hence

\[
\frac{dY}{dX} = -\frac{5}{8}
\]

Once again, in economics we must think of the meaning of these concepts. Here, the sign of the slope is negative. This means that the equivalence scale suggests substitution. If we want more of one good (bread) we must give up some of the other good (eggs) so that we stay within the constraint of the healthy diet. In our case, the slope means that we must give up 5/8th of an egg (Y) for every extra slice of bread (X).

One could say that the ‘health’ price of a slice of bread is 5/8th of an egg!

**Self-assessment**

Before leaving this chapter, check that you can define the following correctly, and give an example of the appropriate form:

- sets and their enumeration
- natural, integer and rational numbers
- planes and xy-coordinates on a plane
- functions, slopes and binding constraints.
Chapter 1: The study of economics

Learning outcomes

At the end of this chapter, you should be able to:

- recall the logic of economic investigation
- define the fundamental economic problem, and describe its immediate derivatives: economic good, scarcity of resources, production possibility frontier and the concept of efficiency, opportunity cost, marginal opportunity cost, desirability, choice and the concept of price opportunity cost.

Reading

LC Chapters 1–2.
BFD Chapters 1–2.

But read this chapter first!

You should read this introductory chapter before you read the introductory chapters in both BFD and LC. (This is an exception to the usual rule for this subject guide.) This chapter will give you a brief and more focused introduction to the study of economics. We will look at some of the underlying issues of knowledge in general, as well as at the fundamental economic problem. You may find the following section somewhat difficult. As we promised in the Introduction, it is therefore optional. You may choose to delve straight into the subject matter of economics, and start reading from Section 2.

Economics as a theory

Note: this preliminary section aims to highlight some basic difficulties concerning economic theory. It is not compulsory and it does not include examinable material. You may choose to skip it and come back to it later. If you do so, please come back to it sometime in the future, as it will widen your understanding of the subject.

What is economics all about?

The most general definition of economics is perhaps this: ‘Economics is the discipline studying the organisation of economic activities in society.’ You may, at first, think that this is too much of an abstraction. After all, how do questions like ‘how much should be produced?’, or ‘what determines prices?’ or ‘how can I make money?’ relate to the general problem of the social organisation of economic activities?

Broadly speaking, particular institutions created by society will have an effect on the answer to the questions posed in the preceding paragraph. The answers will depend, for example, on whether society wishes to have competitive institutions as opposed to, say, cooperative structures. They will also depend on whether decisions are made through a decentralised system of decision-making (which does not necessarily imply competition) or some form of hierarchy. Naturally, the system that will emerge will be a reflection of what is commonly perceived as the ‘economic problem’.
To complicate things, similar institutions may not necessarily reflect similar perceptions of the economic problem. Likewise, similar perceptions may not always produce the same kind of institutions. For instance, Adam Smith was not the first one to point out the benefits of the division of labour. He did so because for him, broadly speaking, the ‘economic problem’ was that of reproduction and growth. He asked how society could organise its activities in order to produce as much surplus (above what is needed for reproduction) as possible.

But forms of division of labour had been recommended before, as solutions to entirely different problems. Plato, for instance, in his *Republic*, suggests a division of labour as a means to create the just society. However, while both of them considered the division of labour as central to the ideal form of social organisation, their institutional recommendations were very different indeed. In part, this can be attributed to a fundamental difference in the way these two scholars understood the world. In a brief and unsatisfactory way one can say that the difference between Plato and Smith is that the former was a kind of ‘rationalist’ while the latter a kind of an ‘empiricist’. Plato felt that the way we know about the world is by the power of our mind. Appearances may be misleading. Smith, on the other hand, wrote in the tradition which followed the principle that knowledge can only be acquired by means of the senses and experience. Consequently, while both of them considered division of labour, the latter attached it to decentralised decision making based on private ownership of property while the former created a clearly hierarchical system with communal ownership among those who make decisions about what society should do, and private ownership among those who provide society with its material wealth.

Put broadly, Smith felt that the division of labour must give rise to the institutions of private property, the market and competition as a means of coordinating economic activities. While Plato felt that division of labour gives rise to communalism – which should not be confused with communism – sharing and cooperation. Evidently, the answers to questions like ‘how much to produce?’, ‘what determines prices?’ and ‘how can I make money?’ are going to be fundamentally different in the two systems. In the end, whatever it is that we are doing, the advice and the recommendations of the economist are all derivatives of the same principles which guide and direct the social organisation of economic activities.

**What is a Theory? The logic of economic investigation**

The world around us seems complex and irregular. Human beings, however, have always been drawn to the idea that there is some order in this apparent chaos. We constantly try and extract order from the world around us, forming a mental ‘model’ of the world. Whether and how such an orderly model relates to the real world are complex questions. At this stage we shall concentrate on how we may create such an order.

Suppose that the political party governing a certain country wants to devise a strategy for re-election. It turns to its analysts to ask for recommendations. In order to advise the party in power, the analysts must find out what makes people vote for the government. They distribute questionnaires, asking people about their general dispositions and economic circumstances. The questionnaires yield two rules:

1. Happy people vote for the government
2. Rich people are happy people.
Notice that these assertions do not have to result from empirical evidence, such as questionnaires. The analysts could have made these statements as **assumptions**: statements that most people would be willing to accept as being true.

To make these assertions, the analysts would need to explain what they mean by ‘happy’ and ‘rich’. Is ‘happy’ a person who jumps up and down for joy at least three times a day, or is it simply someone who is not looking for a new job? Does ‘rich’ mean having a lot of money, though with huge debts to the Mafia, or perhaps having no debts at all? In other words, there is a need to agree on what exactly it is we are talking about. This initial stage of any theory is the **definition** of the subject matters under investigation.

The first phase in building a theory, therefore, is to **define** the relevant components which we **believe** are likely to influence the outcome.

Let us suppose that in some way our analysts clearly define the factors they believe will affect the re-election of the party in power. These factors are: Riches, Happiness, Government and Money. Adding to this our two observations from above, we have the foundation of a theory:

---

**Definitions**

‘Rich people’ (denoted by $R$),

‘Happiness’ (denoted by $H$),

‘Government’ (denoted by $G$),

‘Money’ (denoted by $M$).

**Axioms**

1. ‘Rich people are happy’
2. ‘Happy people vote for the government’

What we now need is a **rule of inference** – a method by which we can enrich our understanding beyond the two axioms. Aristotelian Syllogism is an example of such a rule of inference. It works like this:

<table>
<thead>
<tr>
<th>Premise 1</th>
<th>All humans are mortal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 2</td>
<td>Aristotle is human</td>
</tr>
<tr>
<td>Conclusion</td>
<td>Aristotle is mortal</td>
</tr>
</tbody>
</table>

In our voting analysis, this becomes:

<table>
<thead>
<tr>
<th>Axiom 1</th>
<th>$R$ is $H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axiom 2</td>
<td>$H$ votes $G$</td>
</tr>
<tr>
<td>Conclusion</td>
<td>$R$ (rich people) vote for $G$ (the government)</td>
</tr>
</tbody>
</table>

We call such a conclusion a **theorem**:

**Theorem**

Rich people vote for the government (or, $R$ vote $G$).

This is a **system of logic**. Axioms (premises) plus a rule of inference define a logical system. The conclusions of logical systems are always **logically true**, provided that there has been no mistake in the application of the rule of inference. However, this does not mean that these conclusions are also **empirically true**.
There are, in fact, statements that may be logically true but cannot be confirmed in reality.

A theory produces two types of statements, explanations and predictions. Predictions can be confirmed by some sort of testing. In this case, the theory is verifiable. Explanations, on the other hand, give reasons for empirically observable facts. However, the fact that a theory produces good predictions does not automatically confirm its explanation. In our case, the following propositions can be derived from the above theorem:

**Prediction**
If you give people $M$, they will vote $G$

**Explanation**
People vote $G$ because they have $M$.

In our theory, the prediction is that if we give money to people, the government will win the election. This may be confirmed by observations. We may find that throughout history people were given more money before the election and the party in government had been re-elected. But does this mean that people vote for the government because they have been given more money? Not necessarily.

Suppose now that all elections throughout history took place during spring time. Suppose too that there is a flower called ‘eternum contentum’ which blooms for a short period in the spring, producing a certain special scent in the air which acts like a pacifying drug. Every spring, people act as if they have been collectively intoxicated and are content and happy whatever their circumstances. Can we still say that the empirical truth of our prediction also confirms our explanation? Certainly not.

The problem is that causality is basically not observable. What we normally see are two events occurring in a given sequence. But even if $B$ always comes after $A$, can we say that $A$ causes $B$? Without further information, the answer is that we can not. What we have observed is simply a correlation, a systematic relationship in the occurrence of events. However, both $A$ and $B$ might be caused by some other event $C$, of which we are totally unaware. It is important not to confuse this correlation with causality. This makes the explanatory content of a theory often very difficult to assess.

Naturally, if we believe that the axioms of the theory are empirically true, we may be more inclined to believe the explanations offered by the theory (because we expect the logical structure to carry the empirical truth of the premises over to the propositions). Conversely, if we don’t believe the premises are empirically true, the explanatory side of the theory becomes questionable. Since the goal of a theory is usually its explanatory potential, this can be a problem.

To a great extent, the problem of Normative and Positive economics developed around these questions. Many believe that there are elements in economics which are purely ‘positive’. Namely, that some of the propositions generated by economic analysis are purely descriptive and do not involve any value judgement. For instance, a statement like: ‘increase in demand will raise the price of a good’ seems to be an ‘is’ statement. It describes what is in the real world. Normative economics, taken narrowly, relates to those parts of the theory which are ‘judgemental’. For instance, ‘consumers will be better off when firms have no monopolistic power’ is a normative statement.
However, what exactly is meant by ‘positive’ is highly debatable like the questions surrounding our perceptions and our ability to observe. Generally speaking, people tend to associate an ‘is’ statement with what is ‘positive’. But in our previous example we claimed that ‘demand increases’ when it is not obvious what exactly we mean by ‘demand’. Once we understand what is meant by positivism we shall be able to see immediately what is normative.

Here is an ‘is’ statement:

- ‘John is tall’.

This appears to be a **statement of fact** which would, generally speaking, appear ‘positive’. Still, it isn’t necessarily universally true. Among short people John may be tall, but in a different environment, where the average height is greater, he might not really be considered ‘tall’ at all. Therefore, a truly positive statement would be ‘John is 2.12 metres tall’. This would be universally true (true in all situations), and would not depend on the environment John is in.

Now suppose that the price level in an economy is a **function** of the prices of five goods, and that each good has a different weighting in our price level, depending on the relative amount of spending on that good (the weights will add up to 1). If only one good is purchased and everybody spends their entire income on it, its weight in the price level function would be 1. A good that nobody consumes will have a weight of 0. Let $\alpha_i$ represent the weight of good $i$ ($i$ will be a number between 1 and 5), $P$ be the price level in the economy, and $p_i$ the price of good $i$. Then the price level $P$ is given by:

$$P = \alpha_1 p_1 + \ldots + \alpha_5 p_5$$

where

$$\alpha_1 + \ldots + \alpha_5 = 1$$

A change in the general price level will be the weighted sum of the changes of individual prices in the price index. We denote a change in the price by $dp$. Hence, the general price level will change according to:

$$dP = \alpha_1 dp_1 + \ldots + \alpha_5 dp_5$$

### Example 1

Let the goods and weights be given as follows:

<table>
<thead>
<tr>
<th>Good</th>
<th>Weight ($\alpha_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 bread</td>
<td>0.4</td>
</tr>
<tr>
<td>2 fuel</td>
<td>0.2</td>
</tr>
<tr>
<td>3 transport</td>
<td>0.1</td>
</tr>
<tr>
<td>4 holidays</td>
<td>0.1</td>
</tr>
<tr>
<td>5 health</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Now, suppose that the prices of the various goods change in the following way:

- bread $+20\%$ (hence, $dp_1 = 0.2$)
- fuel $+20\%$ ($dp_2 = 0.2$)
- transport $+10\%$ ($dp_3 = 0.1$)
- holidays $-40\%$ ($dp_4 = -0.4$)
- health $+10\%$ ($dp_5 = 0.1$)

Given the weights in the table, the general price level will then rise by 11%.

- Work out how these individual price increases amount to an 11% increase overall.
How much of an ‘is’ statement will it be if I say ‘Prices have gone up by 11%’? For families who never go on holiday, this will be far from the ‘reality’ of their lives. Moreover, if I now chose to change the weights of the various goods, the statement ‘prices have gone up by 11%’ will simply be incorrect. But the reason it is incorrect is not its failure to describe reality. The problem is that it is describing a subjective reality, based on the consumption pattern of a particular person, which is their choice.

The crucial point here is that a choice was involved in the formulation of this ‘positive’ statement. Each particular weighing system reflects someone’s conception of a good definition of a price level. ‘Price’ here is more an idea than a fact.

It is true that a statement like ‘the price of bread has gone up by 20%’ may appear as a more convincing positive statement. We all know exactly what is meant by the price of bread, and we can observe that it has gone up by 20%. Nevertheless, this is a rather an empty exercise. While it is factually true that the price in terms of money has increased, we must also ensure that we interpret ‘money’ and ‘price’ correctly. Different definitions of these terms might have very different implications for the meaning we associate with a rise in money prices. Since it is this meaning that matters for a positive theory, we again find that a positive theory crucially depends on the choice of definition.

A normative statement is often seen as a statement reflecting a value judgement, for example ‘It is good that the price of bread has gone up by 20%’. The value judgement rests in the definition of ‘good’ and ‘bad’, which clearly makes this a normative, and hence subjective, statement. Value systems are the result of individual choices. However, as we saw above, the conceptual framework giving meaning to the statement ‘the price of bread has gone up by 20%’ is in itself a matter of choice.

This means that the standard distinction between normative and positive statements, which is based on the difference between judgement and description, might be difficult to make in an economic context. Economic theory is full of judgemental descriptions. This implies that we have to be careful when evaluating apparently positive statements, and also that we should not dismiss blatantly normative statements as unscientific. After all, we are all human.

This view of the influence of value judgements on positive economics might seem to dismiss economics as a serious social science. However, nothing could be further from the truth.

It is the strength and beauty of the social sciences that they combine our natural social outlook with the way we form and understand the institutions of society.

**The fundamental economic problem**

The first step in understanding economics is to form an idea of its domain. In other words, what makes something a subject of economic investigation? Alternatively, we can ask what constitutes an economic good.

There may be different answers to these questions which, in turn, will suggest different economic theories. Although it is important to consider different definitions of economic goods, I will concentrate on what is commonly referred to as the Neoclassical interpretation.
**Definition 1**

Everything which is both **scarce** and **desirable** is an **economic good**.

Scarcity is a very straightforward concept: there is a limit to how much of the good is available. Note, however, that scarcity has both a spatial and a temporal aspect. If a good is scarce in one place, while it is not scarce in some other place, it will still be considered an economic good. Similarly, if a good is scarce today, but not likely to be scarce at some point in the future, it is considered an economic good today.

Desirability is a more complex concept. What do we mean when we say we ‘desire’ a good? One might argue that we should distinguish between desiring a good because we **need** it, and desiring a good because we **want** it. The fact that we don’t distinguish between those two sources of desirability is sometimes seen as a defect in neoclassical economics. However, we shall ignore this problem throughout the course.

The third element in the above definition is the emphasis on both scarcity and desirability. Consider the example of fresh air. We clearly need, and hence desire, fresh air to survive. At the same time, air is generally seen as not scarce. Therefore, air would seem to be not an economic good. However, under water, fresh air is clearly scarce, and we are willing to do (and pay) a great deal to have air when we have to go under water. Therefore, air under water is an economic good. Similarly, pollution levels in Paris rose significantly during a recent heatwave. People were willing to refrain from using their cars in order to reduce pollution. Fresh air had become scarce, and hence an economic good. People were willing to pay a price for it by not using their cars.

Try to think of other circumstances in which fresh air might become a scarce, as well as a desirable, commodity.

Leprosy, on the other hand, is scarce but also undesirable. So, leprosy is not an economic good, and there is no price for it.

**Modelling the economic problem**

The definition of economic goods also gives us the definition of the **economic problem**, which is:

- How do we satisfy our desires, or wants, with scarce means?

The definition of the economic problem therefore involves the same ingredients as that of economic goods: scarcity and desirability.

The next step is to ask ourselves what are the implications of this definition. What can we learn from it that we cannot see by simply staring at it? To do so, we must use our common sense until things get too complex. After that we want to use tools which preserve the **logical truth** of our initial intuition. We may not intuitively understand the mechanism of our system but we can rest assured that it carries on the logic of our initial observation. In the end, by looking at what the system yields we may find an explanation which may then appear obvious to us. Nevertheless, this will not make our system redundant as this apparent ‘intuition’ is only reasoning backwards from effects to causes. Constructing such a system is what we call **modelling** and the logical language most commonly used is that of mathematics.
To see what exactly is meant by all this, let us begin by modelling the first component of the definition of economic goods: scarcity. This model is called the production possibility frontier (PPF) or the transformation curve. We shall see that it is the modelling of this basic feature – scarcity – that generates the two most important concepts in economic analysis: price and efficiency.

**The production possibility frontier**

We begin by setting the premises of our model. Consider an economy producing just two commodities, X and Y. In order to produce these commodities, we need a means of production, another economic good which we call labour. Suppose that 1 unit of labour, say one hour of labour, can produce either 2 units of Y or 1 unit of X. (Given that the labour unit (measured in time) is divisible, any linear combination of the two is also possible.) Suppose that there are 100 homogeneous units of labour in the economy.

**Definition 2**

Homogeneity means that all units are identical.

The PPF denotes all combinations of output of X and Y which are possible, given that the labour units are constrained to 100 (which is a direct result of the scarcity of labour), and given the technology in the economy. The technology tells us that each unit can either produce 2 units of Y or 1 unit of X. Given the constraint and the technology, the following table lists some possible allocations of labour to the production of the two goods, and the resulting output of both goods.

<table>
<thead>
<tr>
<th>Labour units in production of X</th>
<th>Labour units in production of Y</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>99</td>
<td>1</td>
<td>198</td>
</tr>
<tr>
<td>99</td>
<td>1</td>
<td>99</td>
<td>2</td>
</tr>
</tbody>
</table>

If all units of labour were engaged in the production of Y, they would have produced 200 units of it (each one can produce 2 units of Y and there are 100 labour units). As each unit can either produce 2Y or 1 X, total production will be 200 units of Y and zero units of X. If, on the other hand, all labour units were engaged in the production of X, they would have produced 100 units of X and zero units of Y. As labour units are homogeneous – which in the present context means that they are of equal ability – we can also allocate some units to X while allocating the rest to Y. If 1 unit of labour is transferred from the production of Y to X, the economy will lose 2 units of Y but gain 1 unit of X, and similarly the reverse is true if we transfer 1 unit from the production of X to Y.

If we assume that units of labour are divisible (so that we can transfer any fraction of labour time from the production of one good to the other) the PPF of the economy will be the one depicted in Figure 1.1 below.
Figure 1.1: The production possibility frontier when output rises in proportion to inputs.

Clearly, we cannot have negative quantities of economic goods. Hence, the space of economic goods is restricted to the positive quadrant, where the values of both \( X \) and \( Y \) are positive. This is the space depicted in Figure 1.1, with \( X \) on the horizontal axis and \( Y \) on the vertical axis. Point \( A \), for example, depicts a bundle containing \( X_0 \) units of commodity \( X \) and \( Y_0 \) units of commodity \( Y \).

Look back to page 13 of the Technical preface if you are not sure what \( X_0, X_1, Y_0 \) etc. represent.

Point \( B \), on the other hand, describes a bundle containing \( X_1 \) units of \( X \) and \( Y_1 \) units of \( Y \). Comparing \( X_0 \) and \( X_1 \), you can see that there are more units of \( X \) in bundle \( A \) than there are in \( B \). Similarly, there are more units of \( Y \) in \( A \) than there are in \( B \). Our desirability assumption means that people will always want to have \( A \) rather than be content with \( B \).

The ‘curve’ (a straight line in this instance) connecting the point where \( Y = 200 \) and the point where \( X = 100 \) is the PPF. The PPF separates what is feasible from what is not. Since neoclassical economics assumes that we desire to have more of everything, the PPF represents a constraint. We can only have those bundles below or on the PPF (that is, in the shaded area or its boundaries).

In Figure 1.1, \( A \) is just feasible, since it is on the PPF. \( B \) is feasible with some units of labour left unused, and hence spare capacity.
We can express the PPF (the curve connecting point \((X = 0, Y = 200)\) with point \((X = 100, Y = 0)\)) algebraically as well. It will have the following form:

1. \(Y = 200 - 2X\)

   When \(X = 0\), \(Y = 200\), and when \(Y = 0\), \(X = 100\).

We can derive this equation by looking at the production constraint in this economy. We know that each labour unit can produce either 2\(Y\) or 1\(X\). In other words, a unit of \(Y\) requires \(\frac{1}{2}\) a unit of labour and a unit of \(X\) requires 1 unit of labour. The total number of labour units available is 100. Thus, the economy can produce any combination of \(X\) and \(Y\) that requires up to 100 units of labour. This constraint on output is expressed in the following way:

2. \(\frac{1}{2}Y + X \leq 100\)

Given that there is a constraint in the number of units of labour available, the PPF denotes the maximum output of \(X\) and \(Y\) feasible for any possible division of labour. Points beyond the curve are not feasible for the economy. This means that if we want more of one of the commodities in our bundle, we have to give up some of the other commodity. We say that the constraint is binding.

The expression on the left of the ‘\(\leq\)’ in equation (2) tells us how many units of labour are necessary to produce a particular level of \(X\) and \(Y\). Any combination of \(X\) and \(Y\) which satisfies (2) is feasible. Thus, (2) defines the production possibility set (which is the shaded area in the Figure 1.1). So a combination of 50 units of \(Y\) and 50 units of \(X\) will satisfy (2) and will be in the feasible set. To produce 50 units of \(Y\) when each unit of \(Y\) requires \(\frac{1}{2}\) a unit of labour means that we will need 25 units of labour. For 50 units of \(X\), we will need 50 units of labour as each \(X\) requires a full unit of labour. Together, therefore, we will need 75 units of labour, which is much less than the 100 units we have at our disposal. On the other hand, a bundle of 100 units of both \(X\) and \(Y\) is not feasible, as you will realise.

---

What is the maximum output of \(X\) we can have together with 60 units of \(Y\), when we have 100 units of labour?

What is the maximum level of \(X\) we can produce if we want 80 units of \(Y\) and when we have 150 units of labour?
Efficiency

Let us now have a closer look at what it means to be on the PPF.

![Figure 1.2: Efficient versus inefficient allocations](image)

At point $B$ we produce 100 units of $Y$ and 25 units of $X$. Equation (2) tells us that this combination is feasible, but does not exhaust all available labour units. The constraint in (2) is: $\frac{1}{2}Y + X \leq 100$. If $Y = 100$ and $X = 25$ the economy is using only 75 units of labour (25 less than what is available). Equation (2) then holds and $B$ is feasible. At point $A$, on the other hand, we produce 150 units of $Y$ and 25 units of $X$. This combination is feasible, and also exhausts all labour units that were available to the economy.

We can draw a horizontal and a vertical line going through point $A$. These lines will separate the space of commodity bundles into four separate quadrants, labelled (i) to (iv).

Is it feasible to locate bundles in each of the four quadrants, and what kind of bundles could they be – for example, how many of the available units of labour would they use? Note down your answers before reading on.

Clearly, producing less of one or both of the commodities in bundle $A$ is always feasible, and will yield a bundle in quadrant (iii). However, we will not be using all available labour at such a point.

Bundles in quadrant (i), on the other hand, are not feasible. If they were feasible, we could have moved from $A$ by producing more of at least one commodity, without having to give up some of the other commodity. This would imply that we hadn’t used all labour resources at $A$, which is obviously not the case.

Bundles in quadrants (ii) and (iv) have more of one commodity, compared to $A$, while having less of the other. This means that to get to a point in, say, quadrant (ii), we have to give up some $Y$ in order to have more $X$. 

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**Note:** The text above is a natural representation of the document content. It maintains the structure and logical flow of the original text, ensuring that all relevant information is accurately conveyed. The use of LaTeX for mathematical expressions and diagrams is simplified for readability and clarity.
How much more $X$ could we have if we gave up 1 unit of $Y$?

Recall that our production technology required one unit of labour for each unit of $X$ and $\frac{1}{2}$ a unit of labour for one unit of $Y$. Hence, an extra unit of $X$ will require that we forgo 2 units of $Y$. Bundles in quadrants (ii) and (iv), where each extra unit of $X$ is associated 2 or less units of $Y$, or each extra unit of $Y$ is associated with $\frac{1}{2}$ a unit of $X$ or less, are all feasible.

We call points like $A$, which require sacrificing one commodity in exchange for more of the other commodity, an **efficient allocation**.

**Definition 3**

An **efficient allocation** of means of production is one which yields a combination of outputs where it is not possible to increase the output of one good without reducing the output of at least one other good.

There are two separate distinct instances of efficiency in an economy, and both will be very important in the chapters that follow. If, as in our example, all economic goods are **tangible goods**, such as food or clothes, the idea of not being able to have more of one economic good without giving up some of another good is called **productive efficiency**.

If, on the other hand, the economic goods include intangible concepts such as the **well-being** of individuals (clearly, both desirable and scarce and thus an economic good), we refer to an efficient allocation as being **allocative efficient**.

**Definition 4**

The set of all **productive efficient** allocations is called the **production possibility frontier**, PPF.

This means that there are infinitely many productive efficient allocations (all the points along the line depicting the PPF). The problem will now be to choose one of these efficient allocations as the most desirable. Once we have found this allocation, we have to investigate which institutional structure (such as competitive markets, cooperatives or planning) will be able to deliver this socially desirable allocation. We can have a first look at the characteristics of such an allocation by conducting a closer examination of the significance of efficiency.

**Opportunity cost**

Suppose that the economy is producing at point $A$. Is there any ‘cost’ associated with this choice? Assuming that we want more of everything, choosing to produce 25 units of $X$ means that we had to give up 50 potentially feasible units of $Y$. Conversely, the production of 150 units of $Y$ ‘cost’ us 75 units of $X$, which we could have produced had we not produced any $Y$ at all. This ‘cost’ which society pays for its choices is called the **opportunity cost**.

**Definition 5**

The **opportunity cost** associated with a particular choice measures how much of the **best possible alternative** had to be given up to make this choice feasible.
This opportunity cost can be interpreted as a general real price. Let us examine the opportunity cost of a unit of $X$. At point $A$, we gave up 50 units of $Y$ in order to be able to produce 25 units of $X$. On average, therefore, the opportunity cost of a unit of $X$ is:

$$\frac{\text{what we had to give up}}{\text{what we got in return}} = \frac{50}{25} = 2 \text{ units of } Y \text{ per unit of } X$$

Write down a similar opportunity cost formula for $Y$.

This opportunity cost, which we can think of as the real price of $X$, will be the same wherever we choose to be on the PPF, since our technology is linear and the labour force is homogeneous. This means that regardless of how much $X$ and $Y$ we produce and regardless of which particular labour unit we use to produce $X$, producing an extra unit of $X$ would require giving up 2 units of $Y$. An extra unit of $X$ always requires the transfer of 1 unit of labour from $Y$ to $X$. This unit of labour could have produced 2 units of $Y$. Hence, the opportunity cost of $X$ will be 2 units of $Y$ per $X$. Notice that this is exactly the slope $\alpha$ of the PPF in Figure 1.2.

Let us review our progress so far by way of theorising. We started with definitions of scarcity, desirability, economic goods, efficiency, the PPF and opportunity cost. The very basic modelling of scarcity has thus produced the following system:

| Premise 1: | All the efficient allocations are on the production possibility frontier. |
| Premise 2: | Only goods produced on the production possibility frontier have an opportunity cost which is greater than zero. |
| Conclusion (theorem): | Only goods which are produced efficiently have an opportunity cost which is greater than zero. |

In the presence of scarcity, there are two kinds of possible allocations of resources. The first kind is an allocation where the feasibility constraint is not binding (i.e. it is not on the PPF, but inside the feasible set). The second kind of allocation is feasible, but the constraint is binding (i.e. we are on the PPF).

Only allocations with a binding feasibility constraint are efficient, meaning that the production of each unit of any good will have an opportunity cost associated with it. Inefficient allocations mean that there is no opportunity cost associated with the production of an extra unit of any good.

There is a paradox here, because we all know that all economic goods seem to have a price associated with them, even when there are clearly productive inefficiencies. Paying the price of a good, which is normally denoted in monetary terms, means giving up something else which we could have purchased with this money. This seems to suggest that there is an opportunity cost associated with all economic goods, regardless of the efficiency of their production. How can we relate this empirical finding notion to the theorem above?
We have to examine the definition of opportunity cost carefully. We defined it as the cost of not using resources for the best alternative production of another good. Recall point \( B \) in Figure 1.2. We produced 100 units of \( Y \) and 25 units of \( X \). From equation (2), we know that this means a total of 75 units of labour. Suppose now that we want an extra unit of \( X \). According to our production technology, we would need 1 unit of labour for 1 unit of \( X \). If we transferred 1 unit of labour from the production of \( Y \) to work on \( X \), we will lose 2 units of \( Y \). It appears that the opportunity cost of 1 unit of \( X \) at \( B \) is 2 units of \( Y \). This clearly contradicts the theorem, which says that only efficient allocations have an opportunity cost associated with them. \( B \), as it is well inside the feasible set, is obviously an inefficient allocation.

Try to work out what the answer to this puzzle is. Concentrate on the full definition of opportunity cost.

Is the opportunity cost of \( X \) at \( B \) really 2 units of \( Y \)? Notice that the definition of opportunity cost refers to those costs which arise from not using resources in their best alternative.

If we choose to employ a worker who is currently producing \( Y \), when there are workers who do not produce anything, we clearly do not use resources in the best alternative. So rather than transferring labour from \( Y \) to \( X \), we should use one of the currently unemployed units of labour to produce more \( X \), leaving the production of \( Y \) unchanged. In this case, the opportunity cost of an extra unit of \( X \) in terms of \( Y \) would clearly be zero.

This does not change the fact that we still have to pay a positive price for a good, regardless of whether the economy is currently producing efficiently. However, we have to differentiate between paying some quantity of \( Y \) for a unit of \( X \) (by using, for example, money, or through some other method of exchange), and paying the opportunity cost.

What’s happening here is that when the price of \( X \) in terms of \( Y \) is greater than the opportunity cost of \( Y \), we are paying more than it really costs to produce \( X \). In such a case, we may say that the economy is inefficient. It means that to get a certain quantity of \( X \) we must produce a sufficient amount of \( Y \) to be able to afford it. But this ‘sufficient’ amount will be more than is really necessary to obtain the quantity of \( X \) we desire. Those resources which are now employed in producing the extra quantity of \( Y \) required for paying for \( X \) could have been used to produce more of both \( X \) and \( Y \).

As both goods are desirable (and we want more of them), any allocation where prices do not reflect the opportunity cost is inefficient. Indeed, one of the most important problems facing economists is how to determine which form of economic organisation will yield prices (or exchange rates) which reflect the real cost of production, the opportunity cost.

**Specialisation and trade**

The main direct conclusion of the way in which we presented the economic problem was that efficiency implies a well-defined cost for the production of each unit of output (the opportunity cost). This, in turn, allows us to examine performance in terms of efficiency by comparing actual prices with opportunity costs. Whenever an economic system produces prices which are the same as the opportunity cost, we may conclude that the system is efficient. If this is not the case, we are paying too much for some goods and too little for others. This, of course, suggests an inefficiency.
But there is another implication which arises from our modelling of scarcity. This is the principle of **specialisation and trade**. If we accept the definition of economic goods as those goods which are both scarce and desirable, and that we have unsatiated wants, then it can be easily established that it is always best for everybody to specialise and trade. Best, here, means that everyone will be able to have more of everything once they specialise and trade. I will demonstrate this point with an example:

Consider two individuals (the heads of households I and II) who produce all their life necessities by themselves. Assume that these necessities include only two types of goods: food \( F \) and clothes \( C \). Individual I can produce, and consume, either 6 units of \( C \) or 2 units of \( F \), or any convex combination of these two extremes.

The circumstances of the second individual (II) allow him to produce, and consume, either 6 units of \( C \) or 6 units of \( F \), or any convex combination of these two extremes. Mapping out the convex combinations of those points will give us the PPF for each household:

![Figure 1.3: Autarky (self-sufficiency): each household consumes exactly what it produces.](image)

Suppose now that the two individuals chose to be at points \( A_I = (1, 3) \) (producing and consuming 1 unit of food and 3 units of clothes) and \( A_{II} = (3, 3) \) respectively. The two individuals, therefore, have organised their households in a productively efficient manner.

Notice that the opportunity cost (or price) of food in household I is 3 units of \( C \) per unit of \( F \) which is the slope of the PPF in the left diagram. In household II, however, the opportunity cost (or price) of food is only 1 unit of \( C \) per unit of \( F \) (the slope of the PPF in the right diagram).

Conversely, the opportunity cost of clothes in household I is 1/3 of a unit of \( F \) per unit of \( C \) while in household II it is 1 unit of \( F \) per unit of \( C \).

We can summarise the position of each household under self-sufficiency as follows:

<table>
<thead>
<tr>
<th></th>
<th>Clothes (C)</th>
<th>Food (F)</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Production</strong></td>
<td>I</td>
<td>II</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td><strong>Consumption</strong></td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td><strong>Opportunity cost</strong></td>
<td>1/3 ( F ) per ( C )</td>
<td>1 ( F ) per ( C )</td>
<td>3 ( C ) per ( F )</td>
</tr>
</tbody>
</table>
Clearly, producing $C$ in household I is cheaper than producing it in household II. This is because the opportunity cost of producing $C$ in household I is only $1/3$ units of $F$ per unit of $C$, compared with 1 unit of $F$ per unit of $C$ in household II. We would say that household I has a **comparative advantage** in the production of $C$. Having a comparative advantage in producing a good means that the opportunity cost of producing that good is lower than it is in other households.

For exactly the same reason, household II has a **comparative advantage** in the production of $F$.

Work out the opportunity cost of a unit of $F$ produced by household II compared with household I.

You can probably see that if every unit of $C$ consumed by both I and II had been produced in household I, while every unit of $F$ consumed had been produced at II, the total amount of $F$ and $C$ would have increased. This could only happen if each household **specialises** in the production of one good, and **trades** with the other to get its preferred consumption bundle.

This is the end of autarky: what each of them produces is no longer necessarily what they will consume. Given the **PPF** of each household, household I will produce 6 units of $C$, while household II will produce 6 units of $F$. Suppose that each household will want to carry on consuming 3 units of $C$. This means that household II will have to buy 3 units of $C$ from I. How many units of $F$ will they be willing to give up to obtain those units of $C$?

We cannot establish the exact price that will emerge at this stage of the course. However, we do know the limits of this price. Household I will not be willing to buy $F$ for more than 3 units of $C$ per unit of $F$, because they could have produced it themselves at that price if they hadn’t specialised. Similarly, household I will not be willing to sell $F$ for less than 1 unit of $C$, because this is their opportunity cost of producing $C$ if they hadn’t specialised.

This means that the **price of a good must be less than its opportunity cost to the buyer, but greater than the opportunity cost to the seller**. Hence:

<table>
<thead>
<tr>
<th>Seller’s opportunity cost</th>
<th>$\leq$</th>
<th>Price</th>
<th>$\leq$</th>
<th>Buyer’s opportunity cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 unit of $C$ per unit of $F$</td>
<td></td>
<td>$P_f$</td>
<td></td>
<td>3 units of $C$ per unit of $F$</td>
</tr>
</tbody>
</table>

The exact price within this range will depend on the **institutions of exchange** and the relative bargaining power of the two households. We will deal with these issues later in the course. At this stage, suppose that the agreed price is 2 units of $C$ per unit of $F$. This exchange rate between $C$ and $F$ is depicted by the line with slope 2 in the two graphs in **Figure 1.4**.
Chapter 1: The study of economics

Both households now have consumption opportunities which they did not have before (the shaded areas in the above figure). This means that trade and specialisation can potentially benefit both households.

If the two households insist on consuming 3 C each, they will now be able to consume more food (1.5 units of F compared to 1 unit of F for I, and 4.5 units of F compared to 3 units of F for II).

Here is a summary of the situation after trade:

<table>
<thead>
<tr>
<th></th>
<th>Clothes (Q)</th>
<th>Food (F)</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>I</td>
<td>II</td>
<td>I</td>
</tr>
<tr>
<td>Consumption</td>
<td>3</td>
<td>3</td>
<td>1.5</td>
</tr>
<tr>
<td>Price</td>
<td>1/2 F per C</td>
<td>1/2 F per C</td>
<td>2 C per F</td>
</tr>
</tbody>
</table>

Evidently, both households are better off (assuming that having more of all goods is indeed equivalent to being better off) after specialising and trading.

The shape of the PPF and the importance of marginal changes

Let us now suppose that labour is not the only means of production. We need machines, as well as labour, to produce both X and Y. To keep things simple, suppose that there is just one kind of machine, and that the use of machines is measured in (homogeneous) machine hours. Let 1 machine hour produce either 1 unit of Y or 2 units of X. Assume a total of 100 machine hours is available.

The production technology with respect to labour is exactly the same as in the previous sections: One unit of labour produces either 1 unit of X or 2 units of Y, and there are 100 labour hours available. However, we cannot produce X or Y by using just one of the means of production. We need both labour and machine time.
According to the new technology we see that in order to produce one unit of X, we would need one unit of labour and \( \frac{1}{2} \) a machine hour. Similarly, to produce one unit of Y requires half a unit of labour and 1 machine hour.

We have now introduced a second constraint which affects our feasible set. As before, we have a labour constraint, the L-constraint, which we have explored above. We found that the feasible set resulting from the labour constraint was given by equation (2):

\[
\frac{1}{2} Y + X \leq 100 \tag{2}
\]

In exactly the same fashion as above, we can derive a similar feasible set from the machine constraint, the M-constraint. It will have the following form:

\[
Y + \frac{1}{2} X \leq 100 \tag{3}
\]

As both labour and machines are needed for the production process of both X and Y, both constraints have to be satisfied simultaneously. This means that a pair of X and Y will be feasible only if equations (2) and (3) are satisfied. Figure 1.5 depicts the space of economic goods with the two constraints:

![Figure 1.5: Introducing a second constraint.](image)

Let us now look at point A again. The combination of 150Y and 25X satisfies equation (2) (i.e. because there is are enough units of labour to produce it). But it does not satisfy equation (3), the machine constraint:

\[
Y + \frac{1}{2} X \leq 100
\]

If \( Y = 150 \) and \( X = 25 \), the left-hand side totals \((150) + (25/2) = 162.5\) machine hours, and since we only have 100 machine hours available, it is no longer feasible to produce this bundle.
So if we insist on producing 25 units of $X$, we will have to reduce our production of $Y$ in order to make it feasible. This means that we will have to reduce $Y$ until we reach the binding limit imposed by the machine constraint. Hence, according to (3), 25 units of $X$ are feasible, provided the production of $Y$ does not exceed 87.5. This is given by point $E$.

Is $E$ in Figure 1.5 a productively efficient point, given that there is labour which is not being employed?

The answer is at the end of the chapter, on page 47. Don’t look at it until you have generated an appropriate answer yourself.

When we have the two constraints (labour time and machine time) the set of feasible allocations becomes the shaded area in Figure 1.6. The overall PPF will now be the kinked line starting at (0, 100) on the $Y$-axis, and going to (100, 0) on the $X$-axis.

Let us now consider the effect of this change in the feasible set on opportunity cost. In Figure 1.6 we consider two points. Point $A$ is at (40, 80) and $B$ is at (80, 40). Both $A$ and $B$ are productive efficient points, lying on the PPF. What is the opportunity cost (or the ‘real price’) of an extra unit of $X$ at points $A$ and $B$?

**Figure 1.6: Marginal opportunity cost.**

At $A$, the overall opportunity cost for the production of 40 units of $X$ is 20 units of $Y$. The opportunity cost of producing one more $X$ can be calculated as the average cost: $20/40 = 1/2$ a unit of $Y$ per unit of $X$. If we now add one unit of $X$ we shall lose 1/2 a unit of $Y$. Here, as in the previous case, the average opportunity cost was a good measure of the marginal opportunity cost. We call the marginal opportunity cost the cost which is associated with the production of the next, or the last, unit of a good. So far we paid little attention to this as the average and the marginal were very much the same. At point $A$ in Figure 1.6 this is still the case. But the importance of special attention to considerations at the margin can be learnt through the examination of point $B$. 
At B, if we follow the same principle (of the average) to derive the opportunity cost per unit we shall get that the cost of one unit of \( X \) at \( B \) is \( 60/80 = 3/4 \) of a unit of \( Y \) per unit of \( X \). This might lead us to believe that the cost of an additional unit of \( X \) will be \( 3/4 \) a unit of \( Y \). However, if we do produce the extra unit of \( X \) we shall find that it had, as a matter of fact, cost us 2 units of \( Y \) rather than \( 3/4 \) of a unit of \( Y \). Namely, the cost of an extra unit depends on how much we are already producing. Instead of the average, we shall have to be more careful and look at what is going on at the margins. The reason for that is the convex shape (towards the origin) of the PPF in Figure 1.6. In Figure 1.2, for instance, we would not have encountered such problems.

So why is the PPF convex towards the origin? The reason that we have discussed above is the existence of multiple constraints. However, even if there was only one factor of production to be considered, the PPF would have been convex had we not assumed the homogeneity of that factor of production. A third reason, not entirely unrelated to the previous ones is the existence of diminishing marginal productivity. We shall discuss the role of the latter reason in more detail as we go along.

How is the slope of the PPF changing with \( X \)? Looking at Figure 1.6, we see that it is relatively ‘flat’ for small values of \( X \), while it is steeper for larger values of \( X \). That is to say, the slope increases with \( X \) in absolute values. We call a curve which exhibits such a property convex to the origin.

Why is the PPF convex to the origin? In the above example, the reason was the existence of multiple constraints. However, we might have a PPF that is convex to the origin even if we only have one constraint, or factor of production. This could be due to some heterogeneity of the factor of production (for example if the skills of some the people providing the labour were greater than those of others). Convexity could also be due to decreasing marginal productivity. We shall explore these reasons further later in the course. However, this shows an important point in the study of economics: This is economics, so we should always try to provide an economic interpretation of mathematical concepts.

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**Self-assessment**

**Check your knowledge**

Check back through the text if you are not sure about any of these.

- Recall the logic of economic investigation.
- Define the fundamental economic problem, and describe its immediate derivatives: economic good, scarcity of resources, production possibility frontier and the concept of efficiency, opportunity cost, marginal opportunity cost, desirability, choice and the concept of price opportunity cost.
- Give an example of the Aristotelian syllogism (rule of inference), homogenous goods and autarky.

**Test your understanding**

In this section, you will find a set of problems of the kind you will meet in the exam. The answers follow on page 44.

If you want to really improve your knowledge, you should try to answer the questions without looking at the answers. After you have answered all
the questions, compare your answers with someone else who is studying this course. If there is no other student you can consult, choose a (patient) friend or family member and try to explain to them the issues involved. It doesn’t matter if they don’t know anything about economics: this will force you to explain the subject in a way that will help you understand things which you would not have understood otherwise. Only after all these trials should you compare your answers with the answers in the book.

**Question 1**
An economy produces two goods, $X$ and $Y$. It uses two means of production, labour and capital. A unit of labour can produce either 1 unit of $X$ or 4 units of $Y$ (or any linear combination of the two). A unit of capital can produce either 4 units of $X$ or 1 unit of $Y$ (or any linear combination of the two). There are 100 units of each means of production.

a. Draw the production possibility frontier of the economy when the two goods can only be produced by a mixture of both factors.

b. What will be the opportunity cost of $X$ if the economy produces 50 units of $X$?

c. Given that the production technology is linear, will the opportunity cost of $X$ remain unchanged when we produce 90 units of it $X$?

**Question 2**
You are still in the economy given in question 1. Suppose that the discovery of new production technologies allows the production of both $X$ and $Y$ by using a single means of production (without a change in their respective productivity).

a. What will the production possibility frontier be now?

b. What will the opportunity cost of producing 50 units of $X$ be? Would it change if we produced 90 units of $X$?

**Question 3**
Robinson Crusoe can bake 10 loaves of bread in one hour or peel 20 potatoes. Friday can bake 5 loaves of bread in an hour or peel 30 potatoes. If they believe in equality in consumption, would they specialise and trade? If so, at what price will they exchange bread for potatoes?

**Question 4**
Developed countries get very little from trade with less developed countries. The reason for this is that all means of production in the developed world are capable of producing much more than any of their counterparts in the less developed countries.

Discuss this statement with reference to the model of specialisation and trade as in question 3.
Answers

Question 1
a. This is a straightforward question which tests your understanding of the principles behind the modelling problem of scarcity. You should have produced a graph like Figure 1.7.

Figure 1.7: Modelling the PPF.

This could have been established by drawing each constraint according to the available information (i.e. 100L can produce either 400Y or 100X, and C can produce either 400X or 100Y).

Alternatively, you could have set up the two constraint equations:

(Labour) $X + \frac{1}{4}Y = 100$

(Capital) $\frac{1}{4}X + Y = 100$

From the symmetry of the model, you can see that the two lines intersect at (80, 80). The PPF is given by the heavy line in Figure 1.7, as both capital and labour (at fixed proportions) are required for the production of each unit of $X$ and $Y$.

b. When the economy produces 50 units of $X$, we are to the left of point $A$ above. The binding constraint is that of capital. Hence, the opportunity cost (the slope of the PPF) is $1/4$ unit of $Y$ per $X$.

c. When we produce 90 units of $X$, we are to the right of point $A$ above. Hence, the opportunity cost of $X$ is 4 units of $Y$ per $X$.

Question 2
The conditions in this question are similar to those in question 1. There is, however, a technological change.

a. We assume now that a change in technology allows us to produce $X$ or $Y$ by using only one of the means of production (capital or labour) at their initial productivity (i.e. 1 unit of $L$ can do either $1X$ or $4Y$, and 1 unit of $C$ can do either $4X$ or $1Y$). If we produce only $Y$ we can produce 400 units by using 100$L$ and an extra 100 by using 100$K$ (500 altogether). When we wish to have $X$ as well we shall first transfer to its production the input which has comparative advantage in producing $X$ (i.e. capital).
Chapter 1: The study of economics

Figure 1.8: The PPF with changed technology.

b. The opportunity cost of $X$ at 50 or 90 will be the same. It will be one quarter a unit of $Y$ per $X$. If, however, we produced 401 units of $X$, the opportunity cost of $X$ becomes 4 units of $Y$ per $X$.

Question 3

The following PPFs should be drawn for Robinson Crusoe (on the left) and for Friday (on the right):

Figure 1.9: Robinson’s and Friday’s PPFs.

Clearly, Robinson has a comparative advantage in baking bread (his opportunity cost for it is 2 potatoes per loaf). Friday has a comparative advantage in potatoes (his opportunity cost for potatoes is 1/6 loaf of bread). Hence, both should specialise and trade with each other.

If they want to be better off from a material point of view as well as pursue other values like equality, the distribution which they should aim for is 5 loaves of bread and 15 potatoes each. Hence, the price of a loaf of bread is 3 potatoes and the price of a single potato is 1/3 of a loaf of bread.
**Question 4**

The crucial step in this question is translating the language of the question into the language of our model.

The question states that developed countries are more productive than less developed countries. In terms of the language of our model, this means that each unit of the input (say, labour) in the developed country can produce more units of either \( X \) or \( Y \) than a unit in the less developed country. This means that the developed country has an **absolute** advantage in production of either good. However, the crucial insight thing is that each country will have a **comparative advantage** in the production of some goods.

In order to translate the question into the language of our model, we have to choose an example which will highlight these features. To keep matters as simple as possible, we assume a world of just two countries, producing two goods. There will be a single input, and its availability and productivity in each country will be chosen to fit the question.

If, say, the developed country can produce 100 units of \( X \) or 100 units of \( Y \), its opportunity cost of producing 1 unit of \( X \) is 1 unit of \( Y \), and **vice versa**.

The less developed country can produce either 40 \( Y \) or 20 \( X \) (it is less developed, and smaller too). Its opportunity cost of producing \( X \) is thus 2 units of \( Y \) per \( X \) (which is more than the opportunity cost of the developed country) but only 1/2 unit of \( X \) per \( Y \) (which is less than that of the developed country). We can then draw the PPF of each country in a diagram. The line connecting (0, 40) and (20, 0) will be the PPF of the less developed country, while that connecting (0, 100) and (100, 0) will be the PPF of the developed country.

![Figure 1.10: The effect of specialisation and trade.](image)

Let us assume that \( Y \) is a luxury good which is of no use to the less developed country at this stage (say a fine malt whisky); \( X \), on the other hand, is an essential good like food.
Before trade, the less developed country will produce and consume as much of the essential good \( X \) as possible. This means it will be at the point \((20, 0)\). Suppose that to begin with the larger economy consumes (and produces) 40 units of \( Y \) and 60 units of \( X \).

Now, allow specialisation and trade. If the larger economy wants to carry on consuming 40 \( Y \), it would be better off buying them from the smaller economy, since the smaller country has a comparative advantage in producing \( Y \). In this case, the larger country can transfer all its means of production to \( X \), where it has a comparative advantage. It can then produce a total of 100 units of \( X \). The larger country will have to transfer part of this to the smaller country to pay for its consumption of \( Y \).

As long as it pays the smaller country more than \( \frac{1}{2} \) \( X \) per \( Y \) (which is what it costs the smaller country to produce \( Y \)), the smaller country will be better off. The larger economy, on the other hand, will not be willing to pay more than \( 1 \) \( X \) per \( Y \), as this is what it would have cost it to produce \( y \) itself. Hence, the price of \( Y \) in terms of \( X \) (\( P_Y \)) can be expressed like this:

\[
\frac{1}{2} \text{ unit of } X \text{ per } Y < P_Y < 1 \text{ unit of } X \text{ per } Y
\]

If the price happens to be, say, \( \frac{3}{4} \) \( X \) per \( Y \) (or \( \frac{4}{3} \) \( Y \) per \( X \)), then the larger economy can will buy its 40 \( Y \) at the price of 30 \( X \), leaving it with 70 \( X \). It will now be able to consume 40 \( Y \) and 70 \( X \), whereas before trade it could only consume 40 \( Y \) and 60 \( X \). The smaller economy will also benefit as well, as it will be able to increase its consumption of the essential good \( X \) by specialising in \( Y \), in which they have it has a comparative advantage. It will now be able to consume 30 units of \( X \) (all of which now come from the large economy) instead of 20.

**Answer to question on page 41**

Is \( E \) a productively efficient point, given that there is labour which is not employed? Yes. This is a good example of the benefits of working with definitions and theory rather than with intuition. According to our definition, a productively efficient allocation is **any allocation where we cannot have more of one tangible good** (a commodity) **without giving up another**. At point \( E \), this is the case. We cannot have more of \( X \) without giving up some \( Y \), because we do not have enough machine hours. Evidently, **productive efficiency** does not imply **allocative efficiency**. Society might well want to choose a point where all means of production are fully employed, but our criterion of productive efficiency is no longer sufficient to guarantee this. Later in the course, we will see what institutional arrangements we need to reach such a point.

**Now read**

- **LC** Chapters 1 to 2.
- **BFD** Chapters 1 and 2.
Chapter 2: Individual choice

Learning outcomes
At the end of this chapter, you should be able to:

- define the concepts of utility, equilibrium price, transitivity, marginal utility, indifference points and indifference curves, income effect, substitution effect, ‘inferior’ and ‘normal’ good completeness and gross substitutes, price elasticity, and real income
- derive utility and indifference curves
- use utility and demand curves to analyse problems involving choice, utility maximisation, substitution and income effects, and price elasticity of demand.

Reading
LC Chapter 5, Chapter 3 pp.38–44 and Chapter 4 pp.65–74 and 76–85.
BFD Chapter 5, Chapter 3 pp.48–49 and Chapter 4 pp.65–82.

The role of demand
Two main issues are normally discussed in the context of consumer’s choice: utility and demand. Before plunging into details let us consider for a moment why are these two concepts so closely related.

One of the most famous illustrations associated with the study of economics is the following:

![Figure 2.1: Supply and demand curves.](image-url)
The vertical axis gives real number values to the price of this good (say, $x$). The horizontal axis gives real number values denoting the quantities of the good. The demand schedule ($D$) depicts the quantity demanded at each possible price while the supply schedule ($S$) relates the quantity that will be supplied at any possible price. There is one point (A) where at a given price the quantity demanded equals the quantity supplied.

Embedded in this picture is a vision of economics which is very similar to the Newtonian vision of mechanics. The world of economic interaction is conceptualised as a world of opposing forces (demand and supply) which are constantly drawn to a balancing point (equilibrium). It is therefore obvious that we would like to examine how each of these forces operates. Utility, in neoclassical economics, provides the explanation to how demand operates.

However, the notion of a downward sloping demand schedule is very old indeed. It is possible to find some evidence of it in Aristotle, St Thomas of Aquinas and certainly among Classical economists like Smith, J.S. Mill and Marx. However, none of the above connected the notion of demand and utility in the same way as we do in neoclassical economics. For many, the downward sloping demand schedule was more like a certainty – like a ‘law’ – than a derivative of a more complex structure. Why then, may you wonder, do we need such a complex structure to derive something which many people seem to agree about anyway?

There are two dimensions to the answer. First, although many people may feel that demand schedules are downward sloping, such a schedule cannot be constructed as an empirical fact. At any point in time we can only establish what people actually do at a given price. If price changes over time, people may act differently for numerous reasons including reasons which are not at all connected to the demand for a particular good. Put differently, to be completely certain that demand schedules are downward sloping, we must be able to observe an individual, or individuals, acting at two points in time where the only thing different is the price. This is obviously impossible. We can try and estimate demand schedules empirically but as a schedule, they do not really exist. Therefore, we cannot be certain that demand schedules are always downward sloping and we must be in a position where we can provide an explanation even if we come across an estimated upward sloping demand. Secondly, there is the question of the usefulness of our theory. As I argued earlier, economics is a language with which we discuss social issues. This means that we cannot only be interested in the prediction power of our theory. We must also be able to interpret situations in a way that will allow us to judge them.

A bridge too far?

For instance, consider the following story. The government considers whether to build a bridge over a certain river. It orders a market research where a demand schedule is being constructed (assume, for simplicity’s sake that the demand was constructed through a questionnaire where people were asked how many times will they use the bridge at different crossing prices). At the same time, it commissions an investigation into the engineering side where it is discovered that given the size of the river, the smallest bridge that could be built is of a capacity for $T$ crossings per day. The graphs overleaf captures these findings:
Chapter 2: Individual choice

Figure 2.2: Demand and supply for bridge crossings per day.

The cost of the smallest bridge is \( C \) but demand and supply do not intersect with a meaningful positive price. Ignoring now the implications of the failure of demand and supply to meet, how can the government – pursuing the interest of the public – form an opinion on whether it is worthwhile building the bridge? If the only use of demand and supply is to predict the price in a market then we will not be able to say anything about whether or not the government should build the bridge. However, if we understood the meaning of the area underneath the demand schedule, we might have been wiser. But to make sense of that area we must derive the demand schedule from a certain construct rather than assume it. We shall come back to this point at the end of the chapter.

Alternatively, consider the following two scenarios:

Figure 2.3: Shifting the demand and supply schedules.

If we merely accepted the downward sloping demand schedule as a premise (or axiom), thus completely disassociating it from utility or any other explanation, we would still be able to predict exactly as we would, had we derived demand from a more complex structure. In the left-hand diagram we can predict that if demand for a commodity increased (a shift to the right of the demand schedule: which means that the quantity demanded at each price will be greater), without any other changes (like, in supply) the new equilibrium price will be higher. But as demonstrated in the right-hand diagram, we would also predict an increase in price...
if supply fell (the supply schedule moves to the left which means that at any price, quantity supplied will be smaller). Considering only these two changes, how can we, as economists, distinguish between these two changes which produce a similar prediction with regard to the price but a different prediction with regard to quantities? Can we judge the one to be in anyway ‘better’ outcome than the other? Can we advise the public – and government – on the social implications of these two changes?

On the face of it, the answer is clear. In the case of an increase in demand, price increased but so did the equilibrium quantity. In the case of a fall in supply the increase in price was accompanied by a fall in the equilibrium quantity. Hence, you may say, the change on the left is ‘better’ than the change in the right-hand diagram.

But this is not so obvious. Let us suppose that the fall in supply resulted from an increase in wages. These would increase the cost of production which means that a seller will sell less at any given price (we shall explore this further in Chapter 3). If you then examine carefully the case of the fall in supply you will find that while there is a fall in equilibrium quantity, there is also an increase in wages. Surely the interest of workers as members of the community cannot be ignored. In addition, it is clear that in the left-hand diagram people buy more of the good but they also spend more on it. What would this mean to the amount of money left for them to spend on other goods? In the right-hand diagram consumers buy less of the good but pay more for every unit. This could mean that they either spend more or less on the good, would it make a difference had it been more rather than less? If you go to a shop and you find that the price of brown rice has gone up and you buy instead the cheaper white rice should this be interpreted as a deterioration in you circumstances? In particular, if at the same time, workers earn more money?

As for the increase in quantity in the case of increased demand, can we say for sure that it is a better sign than the fall in quantity? Suppose that the good in question is a certain fruit: *Nonesensatioalis* which is growing only in one place in the world: the island of *Neverland*. It is considered common food among the indigenous population and there is an equilibrium at point A. One day, it was discovered that the fruit has immense powers of sexual regeneration. All of Hollywood moved to the island and the demand for *Nonesensatioalis* rose. As there are too many rich-and-famous (rafs), the new equilibrium will beat a higher level of both price and quantity. Does this mean that the indigenous population is necessarily better off?

In short, it is difficult to make sense of what the two pictures tell us unless we have further information about what they mean. Clearly one obvious distinction between the two outcomes is that in the case of increased demand, the area underneath both demand and supply increased. In the case of fall in supply, the area underneath demand clearly fell. What exactly happened to the area underneath the supply schedule is unclear. But what does this mean?

Had we only assumed the shape of the demand schedule (as well as that of the supply schedule) we cannot attempt any serious interpretation of the areas underneath both the demand and the supply schedules. We do have an explanation of the outcome (in the one case price increased because of an increase in demand while in the other, price increased because of a fall in supply) but as we have no explanation of what is demand (or supply) we cannot make sense of the outcome.
Introducing utility

The use of utility to explain the demand schedule (as opposed to assuming it) will provide an immediate and coherent interpretation of what the area underneath the demand schedule means. It will also allow us to investigate the relationship between what is happening in one market and the rest of the economic arena. Production functions (or technology) would be equally useful in explaining the supply schedule (as opposed to assuming it) and subsequently, allow us to interpret the area underneath it in a meaningful manner. In such a way, a prediction of an increase in price will have completely different significance when we are able to pour more content into those tools which we feel are the nearest to what can be empirically observed or estimated.

There are many more important implications which can be derived from the way in which we explain those simple tools at the heart of the ‘economics psyche’. We shall later on see that the support for market institutions is very much embedded in the utility interpretation of demand. This means that the study of utility is very important indeed. It will provide a useful means of making sense of economic outcomes as well as provide a justification for a certain kind of organisation for economic activities. At the same time, we must all be conscious of its role as a means of interpretation rather than a confirmed empirical truth.

Rationality

Reading

BFD Chapter 5 pp.92–97.

LC Chapter 5 pp.92–96.

What is rationality?

Modern economics is based on characterising the behaviour of individuals. There is no direct role for more abstract constructs like groups, classes or nations. Economists see these as arising from individual motivation and interaction. Hence, the most important foundation of modern economics is the theory of individual behaviour and motivation.

Individual motivation and desires are very difficult subjects, and many conflicting theories attempt to explain them. Economics has circumvented this problem by asking a slightly different question:

• Given a motivation or desire, how would individuals act to achieve it?

The answer is a simple one: they will choose the best means to achieve that end.

This economic concept of rationality involves two assumptions:

Assumption 1: People know their desires and know the consequences of each choice of means.

Assumption 2: People will behave in a consistent manner. By this we mean that if people have two feasible options available and choose one over the other, they should not, at a later date, choose the other option if both are still feasible.
Consider the following example:

Think about an individual living in isolation. Each season, he can decide how to split up his labour between two goods, $X$ (tomatoes) and $Y$ (cucumbers). If he produces only $X$, he can produce 3 units per season; if he specialises in $Y$, he can produce 6 units of that good. He can also divide his time between the two goods, and produce a combination of $X$ and $Y$. Obviously the combinations that are available to him lie on a line connecting the two extreme points (the solid line in Figure 2.4).

These conditions impose a constraint on what he can consume. Assume now that what our individual is really interested in is . . . a SALAD! The first element of rationality, as we defined it above, would therefore be for him to choose the combination of tomatoes and cucumbers which will produce the most of his favourite salad. He likes both tomatoes and cucumbers but from what is available to him now he prefers point $A$, where he produces 1.5 units of tomatoes ($X$) and 3 units of cucumbers ($Y$), (Figure 2.4).

Why would choosing a point which is not on the constraint be irrational?

By implication, choosing $A$ means that $A$ is preferred over any other available option.

One day, our farmer’s wife gets cross with him and hits him on the head. As a result, our farmer discovers that his abilities have changed greatly. With his labour he can now produce either 4.5 units of tomatoes ($X$) or 4.5 units of cucumbers ($Y$). (This is represented by the dashed line in Figure 2.4.) Assuming that the hit on the head did not affect his tastes, what combination of $X$ and $Y$ should he produce now?

Of course he could remain at point $A$ and carry on producing exactly the same salad as before.
If he moved to a point like B, we would say that our individual was not rational. The reason for this is that he had already made a choice between A and B. When he initially chose to produce A, B (like any other point under the solid line) was equally available to him. By choosing A, the individual is telling us that he prefers A to B. If now he chooses B when A is still feasible, he would be telling us that now he prefers B to A. This means that he is behaving **inconsistently**. Consequently, the only rational options are to stay at A or to move to a point like C (which was not available before) where he has a few cucumbers less but where he is more than compensated for that loss by producing a lot more tomatoes.

A move to C would be consistent because it would mean that he is now choosing a salad combination which is either as good as the one at A or even more to his liking, but which was not available to him before.

Suppose for a moment that our farmer considers the salad at C to be just as tasty as the salad at A. Suppose too, that his wife again hits him on the head and his abilities (but not his tastes) change once more:

![Figure 2.5: Individual rationality in changed conditions.](image)

Now he can produce either 2.5 tomatoes (X) or 7.5 cucumbers (Y). Point A is still feasible. Using the same similar line of reasoning as before, we can say that it would be irrational to move to a point like D (because although this options was available before, he rejected it and chose A instead), but perfectly consistent to move to a point like E where though he consumes fewer tomatoes, he can more than compensate by adding cucumbers.

As before, suppose that he considers the salad at E as tasty as the salad at A (and by implication, as tasty as the salad at C). **Figure 2.6** shows all these developments together.
Figure 2.6: Rationality and revealed preferences.

We can clearly see that the implication of rationality and consistency is that individuals will find points of equal taste arranged along a line like the heavy curve in Figure 2.6. Using this simple implication of rationality, we will now proceed and define this idea more precisely through preferences and utility functions.

Preferences: the relationship individuals have with the world of economic goods

Satisfaction and all that . . .

To analyse the way in which individuals behave when dealing with economic goods (those scarce and desirable things), we must find out how they relate to them.

Example 1

Imagine an old lady with a shopping basket standing in front of the butter and margarine display. In her basket she already has a loaf of wholesome sliced bread. She is trying to decide whether to buy butter or margarine or a bit of both. How will she choose?

In the late eighteenth and nineteenth centuries the school of Utilitarianism was quite prominent in moral philosophy. According to this theory, people derive areal and measurable degree of satisfaction (or ‘happiness’) from their existence, including from their consumption of goods.

According to this belief, our old lady will choose the butter and margarine depending on how much ‘happiness’ or satisfaction they will give her. Perhaps we could find out what she will do simply by measuring her pulse rate! If the idea of eating the entire loaf of wholesome bread with thick layers of butter spread over it raises her pulse from 60 to 80 beats a minute, while the idea of having the same loaf of bread with margarine produces a pulse of only 70, she will probably choose the butter.

The meaning of this is that our choices are based on some measure of gratification. Suppose that next to the old lady stands an old man whose pulse will rise to 100 if he buys the butter, but will stay at 60 if he buys the margarine. Since there is only one pack of butter and one pack of margarine left, we should presumably give the butter to the old man and
the margarine to the old lady. According to utilitarian theories, this would maximise the total amount of happiness created, even if the old lady was not entirely satisfied with this deal.

Work out how much total happiness is created:

a) if the old lady gets the butter and the old man gets the margarine; and
b) if it is the other way round.

If each bundle of economic goods produces measurable degrees of satisfaction, we can easily compare any two individuals and choose a distribution which gives the highest degree of overall satisfaction.

Unfortunately, one of the problems with utilitarianism is that there is no clear way of quantifying different people’s feelings. So economists needed a different way to explain how the old lady makes her choice. The solution was the notion of preferences: if the old lady takes the pack of butter, she is merely indicating that she would rather have wholesome bread with butter than wholesome bread with margarine. So the issue is not one of quantifying her pleasure but rather a question of ranking her preferences.

If we treat the relationship between individuals and the world of economic goods as a matter of ranking (idea being that when an individual is confronted with two bundles \( A \) and \( B \) of goods, they will always say either ‘I prefer \( A \) to \( B \)’, ‘I prefer \( B \) to \( A \)’ or, ‘I like \( A \) and \( B \) equally’) we can consider a much broader set of motivations. This, in principle, lends the theory an important degree of generality which is much more appealing than the narrow and intellectually unacceptable notion of measurable satisfaction.

Representing preferences

For the purpose of analysing the relationship which individuals have with the world of economic goods, we may wish to begin with a more straightforward and descriptive instrument. We may want to describe what people might say when confronted with at least two bundles of economic goods.

Let \( A \) and \( B \) be such bundles. An individual is bound to say either ‘I prefer \( A \) to \( B \)’ or ‘I prefer \( B \) to \( A \)’ or ‘I like \( A \) and \( B \) equally’. Let us denote these three possible statements by the following preference symbols:

- ‘\( A \succ B \)’ means ‘\( A \) is preferred to \( B \)’
- ‘\( A \sim B \)’ means ‘indifferent between \( A \) and \( B \)’.

This depiction of the attitude which people might have towards the world of economic goods generates a great deal of analytical difficulty. It is true that most of the time, people will be confronted with binary choices (like the one between \( A \) and \( B \)). But what concerns us is not only the single choice of a single individual but the simultaneous choices made by many.

To that end, we must be aware of what our individual would do had they confronted a different choice. If, say, a child is offered a choice between a Train Set (\( TS \)) and the game Snakes and Ladders (\( SL \)) she might choose \( TS \) (which means that she prefers \( TS \) to \( SL \)). However, when offered a choice between \( TS \) and a Pottery Wheel (\( PW \)) she might choose \( PW \) (which means that she prefers \( PW \) to \( TS \)). If now she is being offered a choice between \( PW \) and \( SL \) she might choose \( SL \), which means that she prefers \( SL \) to \( PW \).
Write down the girl’s preferences using preference notation (A > B et cetera).

What have we got here? TS is preferred to SL (TS > SL); PW is preferred to TS (PW > TS) and SL is preferred to PW (SL > PW). This means that if the child goes into a toyshop where she is confronted with all the goods at once she will have a logical problem: PW > TS > SL > PW. Which toy will she choose?

In other words, it is not sufficient to ask the individual to rank only two bundles, we need to know their preferences with regard to all the other bundles. That is, we want a complete ordering (ranking) over the whole space of economic goods. However, people are highly unlikely to have such a comprehensive knowledge of their preferences. We must therefore move from this literal description of people’s attitude towards economic goods towards a more abstract depiction of these attitudes.

Not surprisingly, moving from the simple binary choice problems to a more abstract depiction of attitudes creates problems of its own, as we saw in the case of the child having to choose between three goods. To resolve this and other issues, we will have to make some assumptions about individuals’ preferences.

The two most crucial assumptions involved here are those of completeness and of transitivity. We touched on completeness in the example above: it is the requirement for individuals to be able to give a complete ranking of all bundles available to them, from most preferred to least preferred. Completeness is both a technical and substantive assumption. Technical because it is required for the presentation of preferences and substantive because it is a departure from the description of indifferent behaviour. On the one hand it is an important assumption which enables us to use a continuous, real number function to represent preferences (the utility function, more of which later). On the other hand, it is a logical extension of the idea of individual choice. We are always able to rank two or more things in order of preference, but in most cases, we are not choosing between one thing or the other, but between complex bundles of goods. The ranking of such bundles is a much more delicate and complicated issue. We are necessarily assuming that our economic individuals are able to perform such rankings.

The assumption of transitivity, in spite of having important technical implications, is first and foremost a substantive one. Transitivity is one way of introducing rationality into our analysis. By assuming that preferences are transitive we exclude the possibility of the above-child’s predicament. Our child’s preferences PW > TS > SL > PW are inconsistent, since PW is apparently preferred to itself. Had her preferences been consistent, they would have satisfied transitivity: PW > TS > SL. This is what would have occurred if the child had been offered a binary choice between PW and SL: her preferences would have been PW > SL. Preferences, one might say, must reflect consistency. (This, one may argue, is the most fundamental principle of rationality.)

To remind you, we abstracted from the simple depiction of what people might say when offered a choice between two bundles, because it doesn’t allow us analytically to gain much insight. Our goal is to move to a more powerful instrument, which is also easier to handle. One such instrument is a real number function. This may be less intuitively representative of the world of preferences, but real numbers are much easier to use. In order to make the transition from preferences, which we denoted by the preference sign > to real number functions, we need the completeness...
and **transitivity** assumptions, plus a few more technical assumptions. Since we end up with a real number function, we can now replace the preference sign with the more familiar ‘greater than’ (>) sign.

The transition works like this: Consider a world of two economic goods, X and Y. The space of all possible bundles of economic goods is the positive quadrant of the plane, as shown in **Figure 2.7**:

**Figure 2.7: The consumption space.**

A point like A depicts a bundle which consists of \(X_0\) units of X and \(Y_0\) units of Y. We write \(A = (X_0, Y_0)\). Similarly, B is a point where we have \(X_1\) units of X and \(Y_1\) units of Y (\(B = (X_1, Y_1)\)). The subscripts and so on are ways of identifying different ‘packages’ of X and Y. They do not indicate the magnitude of X and Y.

We can work the process through like this:

1. Write down what people might actually say, for example: ‘I prefer A to B’.
2. Write this down using preferences symbols: in this case \(A > B\) (meaning: A is preferred by individual \(i\) to B).
3. Introduce the concept of **weak preference**: For two bundles A and C, \(A \succeq C\) means ‘I prefer or am indifferent between A and C’. This is actually a very small extension to stage 2 above, but makes the transition to a real number function much easier to make.
4. Extend the ordering ‘>’ over the entire set of economic goods (so that each individual can at all times rank all possible bundles). This means that our completeness assumption must hold.
5. Assume that the above ranking is rational and therefore satisfies transitivity.
6. We have now ranked all available consumption bundles in order of weak preference: say, \(A \succeq B \succeq C\). Now, we assign a real number to each of these choices, with the property that numbers assigned to weakly preferred choices are weakly bigger than those assigned to non-preferred choices. Denote the number by \(U\) (for utility). Then, we clearly have \(U(A) \geq U(B) \geq U(C)\). We are thus **mapping** from preferences onto real numbers. We call this mapping the **ordinal utility function**. **Ordinal** means that the function only tells us about
the order, or ranking, of the bundles. The magnitude of the numbers has no significance.

Example 2
Consider two individuals 1 and 2, with \( U_1 \) and \( U_2 \) as their respective utility functions. If \( U_1(A) = 1000 \) and \( U_1(B) = 20 \) we know that individual 1 prefers A to B \( (U_1(A) = 1000 > U_1(B) = 20) \); if \( U_2(A) = 100 \) and \( U_2(B) = 20 \) we know that individual 2 also prefers A to B. Remember that the actual numbers have no extra significance, it is only the inequalities that matter.

- If this is so, and if individual 1 has B and individual 2 has A, can we say that it is desirable to ask individual 1 and 2 to swap their bundles?

A utilitarian might want such a swap on the grounds society should maximise the total amount of utility. On this argument, the present allocation of A to 2 and B to 1 gives us a total utility of \( U_1(B) + U_2(A) = 20 + 100 = 120 \). But if we give B to 2 and A to 1 the sum would change to \( U_1(A) + U_2(B) = 1000 + 20 = 1020 \). However, the magnitude of these numbers means nothing whatsoever. The fact that 1’s preferences are represented by 1000 against 20 and 2’s preferences are represented by 100 to 20 is insignificant. However, we cannot say such a thing. The only thing we can say is that both individuals prefer A to B.

On the other hand, when we choose to allow numbers to represent the strength of our preferences we have a different utility function which is more than just a ranking: this function is called **cardinal utility**. Naturally, in such a case, the comparison of utilities between the two individuals would have been meaningful.

Properties of utility functions
The ordinal utility function, then, simply represents individuals’ preferences over the space of economic goods. Let us now examine the properties of the utility function that represent those preferences.

We will begin by looking at a point like A in **Figure 2.8**. From what we have said so far about preferences, A immediately defines 4 quadrants around the horizontal (\( X \)) and vertical (\( Y \)) axes in this graph:

![Figure 2.8: Preferences.](image-url)
Chapter 2: Individual choice

- Clearly A is preferred over all points in quadrant III \((A \succ B)\), because at A, we have more of both goods, which must be at least as good as being at B.
- All points in quadrant I are preferred over A \((C \succ A)\).
- The line connecting B and C goes through either quadrant II or quadrant IV. Therefore, as we move from an inferior point (B) to a superior one (C), we must go through a point where we are indifferent between the two bundles (point D). All such points must be either in quadrant II or IV.

As we explained above, a function \(u\) is said to represent these preferences if \(A \succ B\) implies that \(U(A) \geq U(B)\). What other properties must such a utility function possess? Remember that each bundle, such as A, actually consists of certain amounts of various goods. In the graph above bundle A consists of \(X_0\) units of good \(X\), and \(Y_0\) units of good \(Y\). We can then show that:
- \(u\) must be increasing in both \(X\) and \(Y\): the more we have of either good, the more preferred the bundle is.

Marginal utility

For a given level of, say, \(Y\), we can define the marginal utility of good \(X\) \((MU_X)\). Mathematically, this is defined as:

\[
MU_X(Y_0) = \frac{dU}{dX} \bigg|_{X = X_0}
\]

This construct is called the derivative of \(U\) with respect to \(X\), keeping \(Y\) constant at \(Y_0\). It tells us how utility would change if we changed \(X\), while keeping \(Y\) constant.

The form which we give to the utility function reflects our beliefs about how people relate to the world of economic goods (i.e. their preferences). Having defined the utility function, we can look at its implications.

Example 3

Assume that the consumption bundles among which our individuals have to choose consist of two goods, bedrooms (A) and television sets (B). Let us say that the number of bedrooms is 3. You will probably agree that a change from 0 television sets to 1 television set represents a substantial increase in utility. On the other hand, if we already have 14 television sets and add a fifteenth one, the increase in utility is likely to be insignificant.

There are two elements to this story:
1. increases in utility as we increase consumption of one good (marginal utility) will depend on how many units of that good we already consume
2. the marginal utility will depend on how much of the other goods we consume (if we have 30 bedrooms, the fifteenth television set might come in quite handy...).

This example points to a more general property of utility functions, as they are typically defined in economics: diminishing marginal utility. For a given consumption of all other goods, utility will rise in diminishing increments with an increase in one particular good.
Marginal utility, as we have shown above, corresponds to the derivative of the utility function. The derivative of a function is the change in the function's value for a given change in one of its variables. This, of course, is called the slope of a function.

Consider Figure 2.9. It shows utility on the vertical axis, and the consumption of $X$ on the horizontal axis. The consumption of the second good, $Y$, is held constant at $Y_0$. Initially, we are consuming $X_0$ units of good $X$ and $Y_0$ units of good $Y$. If we now change our consumption of $X$ by an amount $dX$, utility would change by $dU$. The magnitude of $dU$ will depend on where the initial $X_0$ is located.

How would the magnitude of $dU$ change as we increase the initial $X_0$?

Had the initial $X$ been at $X_1$, $dU$ would have been much greater.

Note that there is a slight problem with the concept of marginal utility: A little while ago, we said that utility should be viewed as ordinal, and that the actual numerical values did not matter. What, then, is the significance of the falling marginal utility? This issue would not arise if we viewed utility numbers as actually depicting the intensity of an individual’s preferences.

**Indifference points**

As seen in Figure 2.10, when moving from a point such as $B$, which is inferior to $A$, to a point such as $C$, which is preferred to $A$, we must pass through a point $D$, where we have no preference between $D$ and $A$. We are indifferent between bundles $A$ and $D$.

The points which we rank as equally preferable to $A$ will lie on a downwards sloping line, going through $A$ (by definition, we are indifferent between a bundle and itself) and $D$. Such a line must go through quadrants II and IV.

The reason why we can make such an assertion is our assumption about the continuity of preferences. That is, if we take the line connecting point $B$ with $C$, we can see that there are many bundles along it. We know that at $B$, $U(B) < U(A)$ and at $C$, $U(C) > U(A)$.
Let $\alpha$, on the horizontal axis of Figure 2.11, be a bundle on the line between $B$ and $C$. On the left-hand side of the graph, $\alpha = B$; at the other end of the diagram, $\alpha = C$. On the vertical axis, we can write the difference in utility between point $A$ and point $\alpha$: $U(A) - U(\alpha)$. This is positive on the left-hand side, since $U(A) > U(B)$, but negative on the right-hand side, since $U(A) < U(C)$. As we move gradually from $B$ to $C$, we move from a less preferred to a more preferred bundle. On our way, we must cross the point where $U(A) - U(\alpha) = 0$. This is the point where $\alpha = D$ and it means that $U(A) = U(\alpha)$ or, that the individual is indifferent between $A$ and $D$. Ordinality is trivial here because both points $A$ and $D$ give the same utility to the consumer.

**The slope of the indifference curve**

There are many indifference points in Figure 2.12, where the utility of the bundle at that point is the same as the utility of point $A$. We now want to characterise the location of all such points, and draw a curve connecting them.

Consider point $A$ again. If we give up one unit of good $X$, we shall lose the utility of that unit, as specified by the marginal utility $MU_x$. Hence, the total change in utility will be $-dX \cdot MU_x$. If we change our consumption of $Y$ at the same time by an amount $dY$, our change in utility from that will be $dY \cdot MU_y$. 

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**Figure 2.10: Indifference points.**

**Figure 2.11: Differences in utility for different bundles.**
Therefore, if we give up $X$ and increase $Y$, there will be a point where our loss of utility from $X$ will be fully compensated by the increased utility from $Y$. At that point,

$$\frac{dX \cdot MU_X}{dY} = \frac{dY \cdot MU_Y}{dX}$$

We know from geometry that the slope of a curve, in particular the indifference curve, is given by $-(dY/dX)$. Hence, rearranging the above equation, we find that the slope of the indifference curve is given at $A$ by:

$$\frac{dY}{dX} = \frac{MU_X}{MU_Y}$$

What does this equation mean? $MU_X/MU_Y$ will be a number, say 5. What does this number represent? The answer to this is crucial, and you should bear it in mind at all times. Say $MU_X = 10$ and $MU_Y = 2$. Then the fact that $10/2 = 5$ means that the individual would be willing to give up 5 units of $Y$ in exchange for 1 unit of $X$. As we have seen, by construction, she will be neither better nor worse off as a result of this change.

In other words, the slope of the indifference curve at any point represents the individual’s **willingness to pay** for $X$ in terms of $Y$, in other words how much of $Y$ he would be willing to give up in exchange for one extra unit of $X$. It is also referred to as the MRS: the **marginal rate of subjective substitution**. This means the same thing: if we were to substitute the consumption of $X$ for the consumption of $Y$, it refers to the number of units of $Y$ we could take away for an extra unit of $X$.

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**Figure 2.12: The slope of the indifference curve.**

Going back to the cucumbers/tomatoes example in **Figure 2.5**, what are the three different MRSs illustrated?
Consider the points \( A \) and \( B \) in Figure 2.9, which lie on the same indifference curve. As we mentioned above, the slope of the indifference curve represents an individual's willingness to pay at a particular point.

At point \( A \), the individual has only a few \( X \), while she has plenty of \( Y \). Surely, at this point, \( X \) (being scarcer) is more precious to her than \( Y \) (think back to our example of the bedrooms and television sets), and she would be willing to give up quite a lot of \( Y \) in exchange for one more \( X \). Consequently, the slope of the indifference curve at \( A \) (the number of \( Y \) the individual would be willing to give up for one more \( X \)) is very steep. At \( B \), on the other hand, our individual has plenty of \( X \) and only a few \( Y \), so her willingness to pay for one more \( X \) in terms of \( Y \) should be much lower: The slope of the indifference curve is much flatter.

Such an indifference curve is called *convex*. As we move along the indifference curve from the top left (where we have a lot of \( Y \) but little \( X \)) to the bottom right (where the opposite is true), the marginal utility of \( X \) will decrease (\( \downarrow \)), while that of \( Y \) will increase (\( \uparrow \)):

\[
\left( \frac{\downarrow MU_X}{\uparrow MU_Y} \right) \downarrow
\]

This is mainly a result of the decreasing marginal utility of consumption, as we discussed above.

**Individual behaviour and the budget constraint**

**Reading**

- **BFD** Chapter 5 pp. 103–16.
- **LC** Chapter 5 pp. 97–100.

We have now defined what we mean by the desirability of economic goods through the concept of utility functions. We now have to complete the picture by adding scarcity to our world picture.

We introduce scarcity through the concept of a budget constraint. Let an individual have a money income \( I \). In her world, there are only two goods, \( X \) and \( Y \), and she has to choose a bundle consisting of those two goods. If we denote the prices of goods \( X \) and \( Y \) by \( P_X \) and \( P_Y \) respectively, the
individual can choose bundles \((X, Y)\) such that the cost of those bundles is at most \(I\), the total of her income:
\[
P_X X + P_Y Y \leq I
\]
We call this the budget constraint of the individual: her choice of bundles is constrained by her income, or the budget she has available for consumption. Clearly, if she chooses a bundle such that the inequality above is strict \((<)\), she will have some money left over. The curve connecting all the bundles where she spends all her income (that is, when the equation above holds with equality, \((=)\)) is called the budget line.

The budget line, a bit like the production possibility curve, divides the world of economic goods into what is possible and what is not. The intercepts with the horizontal and vertical axis are the points where the individual uses their entire income for the consumption of one good. In such a case, the individual will be able to buy \(I/P_i\) units of good \(i\) (where \(i = X, Y\)).

![Figure 2.14: The budget line.](image)

The slope of the budget line reveals yet another concept of exchange. Recall that so far we have talked about two such concepts. First there was the slope of the production possibility curve which represented the opportunity cost, or the technological rate of substitution. As technology is assumed to be given, this exchange rate between \(X\) and \(Y\) represents the social cost. It tells us how many units of \(Y\) (or \(X\)) we really need to give up in order to obtain one more unit of \(X\) (or \(Y\)).

The second concept of price, or exchange rate, was the subjective rate of substitution, or what one is willing to pay for one unit of \(X\) (and \(Y\)). This exchange rate was entirely dependent on individuals’ preferences.

Now we have the slope of the budget line which will give us the market rate of exchange between \(X\) and \(Y\), or the price of \(X\) in terms of \(Y\) (and also the price of \(Y\) in terms of \(X\)). If we are consuming \(X\) and \(Y\) such that our income is exhausted, we are said to be on the budget line. The total spending on \(X\) \((P_X X)\) plus the total spending on \(Y\) \((P_Y Y)\) equals our income \((I)\) (for instance point \(A\) in the above diagram).

If we now choose to consume 1 less unit of \(X\) \((dX = 1)\), how many more units of \(Y\) will we be able to buy? If \(P_X = £10\) and \(P_Y = £5\), then giving up one unit of \(X\) will leave £10 which we can now spend on \(Y\). Given that the
price of Y is £5, we will be able to buy 2 units of Y (denoting the slope of the budget line by \( \alpha \), \( dY = \alpha dX \)). In our case, the slope is: \( \alpha = \frac{P_X}{P_Y} = \frac{10}{5} = 2 \), hence \( dY = 2 \). Therefore, the slope of the budget line reveals the exchange rate between \( X \) and \( Y \) that will be available in the market.

**Utility maximisation**

**Reading**

BFD Chapter 5 pp.119–21.

Assuming that the individual always wants more of all economic goods, they would want to choose the most preferred bundle from the set of feasible bundles. We know that utility is increasing in both \( X \) and \( Y \) (see above). We also know that each level of utility corresponds to a convex indifference curve. Translating this into the language of the model we say that the individual wants to maximise utility (i.e. to choose the most preferred bundle) subject to the budget constraint (i.e. from the set of feasible bundles that they can afford). Graphically it means to choose the highest indifference curve possible.

![Figure 2.15: Utility maximisation.](image)

Given the shape of the indifference curve, the highest level of utility will be achieved whenever the indifference curve is just tangent to the budget line. This is because a higher indifference curve denotes a higher level of utility. However, an individual has to be confined within their budget and thus the maximum utility that can be attained is at the point where the indifference curve is just tangent to the individual’s budget line.

Note that ‘tangency’ means that the slope of the indifference curve is the same as the slope of the budget line at the point of tangency. So a consumer chooses the optimal consumption bundle whenever their subjective rate of substitution \( \frac{MU_X}{MU_Y} \) equals the market rate of exchange \( \frac{P_X}{P_Y} \). In other words, the individual pays for a unit of \( X \) in the market place exactly as much as they are willing to pay!

The utility maximising individual will therefore want to consume \( X_0 \) of \( X \) and \( Y_0 \) of \( Y \) (point A in Figure 2.15 in the world of two economic goods. A point like A represents an optimal choice because there is nothing the individual can do – within this framework – that will bring a higher level of utility (or a more preferred bundle). At A, the subjective rate
of substitution \((MU_x/MU_y)\) is the same as the market rate of exchange between X and Y, \(P_x/P_y\).

Figure 2.16: Suboptimal bundles.

If the individual is at a point like B, the subjective rate of substitution is lower than the market rate of exchange. The slope of the indifference curve \((MU_x/MU_y)\) is smaller than the slope of the budget constraint \((P_x/P_y)\). This means that at B if the individual gives up one unit of X, they would need \(\alpha\) units of Y to regain the same level of utility as B. However, if they do give up one unit of X, they can afford \(\gamma = \alpha + \beta\) units of Y per unit of X at market prices. This means that they will be better off exchanging some X for Y. This will be true as long as the subjective rate of substitution differs from the market rate of exchange. In technical terms, they will exchange X for Y as long as the slope of the indifference curve differs from the slope of the budget constraint.

At A, however, the subjective rate of exchange equals the market rate of exchange. Hence, if the individual gives up one unit of X at A, they will need more than \(\gamma\) units of Y per unit of X to be able to increase utility. However, in the market place they will get precisely \(\gamma\) units of Y per unit of X. Therefore, if their aim is to increase their utility, they will not change their consumption bundle once they get to a point like A. Therefore, A is the point where they maximise their utility given the budget constraint.

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**Deriving demand for economic goods**

**Substitution and income effects**

**Reading**

- **BFD** Chapter 5 pp.113–16.
- **LC** Chapter 5 pp.101–05.

Having explained how individuals are making their choices, we can now establish the downward sloping demand curve in the plane of quantity and price as a result of utility maximisation. Figure 2.17 depicts this analysis:
Chapter 2: Individual choice

Figure 2.17: Utility maximisation and the demand function.

At A, the price of X is given as \( P^0_X \) and, for a given price of Y (\( P^0_Y \)) and a given income \( I_0 \), the quantity of X that will be demanded is \( X_0 \). To analyse the impact of a change in the price of X alone on the quantity demanded we have to keep the price of Y, as well as income, unchanged. We simply set \( P_1^X (\lt P^0_X) \) as the new price and repeat the analysis above to reach point B as the new optimal choice. At B, clearly, the quantity of X demanded is greater than before. Plotting the price of good X against the demand for good X on the right-hand side, we observe a negative relationship between the two: The lower the price, the higher is the quantity demanded when the individual maximises utility. The downward sloping demand curve is now a conclusion rather than an assumption.

But now we have further insights into these changes. We can see that a fall in the price of X will shift the budget constraint in Figure 2.18.

Figure 2.18: Changing the price of good X.

The shaded area represents a whole new range of opportunities (either to consume more X or to transfer some spending to Y) which were not feasible before. Thus, without a change in the individual’s nominal income, there seems to be a rise in their real income: there is an income effect. Given that individuals find both the two goods desirable, a change in the
price of $X$ (which brings about a change in real income) will generate a change in the demand for $Y$ too. We therefore have to ask how the effects of the rise in the price of $X$ will be distributed between the two goods.

Also, if the change in the price of $X$ has an income effect, then many government policies that change the price we pay for goods (like taxation and subsidies) will make people feel either richer or poorer in real terms. Assuming that such policies are also concerned with redistribution of income, this is an important new insight that we could not have derived by treating the downward sloping demand curve as a ‘law’.

At the same time, since individuals find both goods desirable, and can get the same level of utility consuming different proportions of both goods, we will also observe a substitution effect. Individuals are likely to substitute away from the good that is now relatively more expensive ($Y$ in our example) to the one that is now relatively cheaper ($X$).

Can we distinguish the substitution effect from the income effect (in other words between that much more of $X$ that we buy because it is cheaper now, and that much more of it that we buy because we feel richer)? Yes, but first we must establish what we mean by real income. Look again at Figure 2.18: it is evident that in terms of $X$ alone there was a significant rise in real income but in terms of $Y$ alone there was no real rise at all. So has real income gone up or not?

We will consider two approaches to this question. One, following Hicks, suggests that the relevant measure for real income is utility. The other, following Slutsky, points to the initial bundle as the reference point for real income. Let us consider those two approaches in turn.

**Hicksian income and substitution effects**

According to Hicks, real income is measured in terms of utility. Hence, the substitution effect can be established by looking at the individual’s optimal choice had they confronted the new relative price (the new exchange rate between $X$ and $Y$), while keeping utility at the initial level $U_0$. This can be achieved by finding the point of tangency of the new relative price with the initial indifference curve, $U_0$ (point $C$ in Figure 2.19):

![Figure 2.19: The Hicks substitution and income effects.](image)

The move from $A$ to $C$ is what we may call the pure (or, sometimes, net) substitution effect. It simply tells us how a utility-maximising individual
would respond to a new market exchange rate between $X$ and $Y$ if their real income remained unchanged. Given the Hicksian definition of real income, based on utility, the individual enjoys the **same level of real income** at both $A$ and $C$, because their utility is the same at these two points.

Notice that the move from $A$ to $C$ is determined by the shape of the indifference curve (which is the same as the utility curve). The reason why an individual will consume more of $X$ as the price of $X$ (in terms of $Y$) falls is that the market price at $A$ for $X$ is now less than they are willing to pay for it. Evidently, the optimal behaviour for this person in such circumstances is to buy more of $X$.

As the individual buys more of $X$ (and also consumes less of $Y$), the marginal utility of $X$ will decrease while the marginal utility of $Y$ will increase. $MU_x/MU_y$ is now smaller: as we have plenty of $X$ and only a few $Y$, the willingness to pay for $X$ will be reduced until the individual gets to the point where her willingness to pay is the same as what she is being **required** to pay in the market place.

Hence, due to the convexity of the indifference curve, there is always an inverse net-substitution relationship. In other words, because of diminishing marginal utilities we will buy more of the good whose price has fallen.

Let us now consider the move from $C$ to $B$. Naturally, as both $A$ and $C$ are on the same indifference curve (meaning that they are at the same utility level) and at $B$ we are on a higher level of utility, the move from $C$ to $B$ must be the income effect.

The move from $C$ to $B$ in Figure 2.19 can be brought about by a parallel shift of the budget line caused by an increase in nominal income between points $C$ and $B$. However, there has been no change in nominal income, so budget lines at $B$ and at $A$ are for the same level of nominal income. Evidently, then, at $C$ the income which can be associated with the initial real income ($U_0$) must be lower than the income at $A$. The difference between income at $C$, (the amount of money needed to sustain the original level of real income at the new prices) and income at $A$ can be considered as a **nominal equivalent to the real income effect**.

For instance, let $I_0 = 100$, $P^0_X = 10$ and $P^0_Y = 10$. Point $A$ in Figure 2.20 captures the initial position where the consumer chooses the bundle (5, 5).

![Figure 2.20: Hicksian income and substitution effects.](image-url)
Now the price of $X$ has changed to $P_X^1 = 5$. The individual will move to a new preferred choice – point $B$ above, where she consumes $(8, 6)$. Following Hicks, we want to isolate the substitution effect by looking at what the individual would have chosen if she confronted the new relative price at the original level of utility. Point $C$ in Figure 2.20 is such a point, where the individual’s choice is, say, $(6, 4.2)$. We can now calculate the level of nominal income that would have been needed for her to be at point $C$.

\[
\begin{align*}
A & \quad P_X^0 X_0 + P_Y^0 Y_0 = I_0 \\
& \quad 10 \times 5 + 10 \times 5 = 100 \\
C & \quad P_X^1 X_C + P_Y^0 Y_C = I_C \\
& \quad 5 \times 6 + 10 \times 4.2 = 72
\end{align*}
\]

So the shift from $C$ to $B$ can be explained as an equivalent to a rise of 28 in income (from 70 to 100), if there was no substitution effect to be considered (that is, if relative prices had not changed). Notice, however, that there was no actual change in nominal income during the move from $A$ to $B$.

You may also have noticed that in this case, at point $C$ the individual could not consume the bundle which is depicted by $A$: the budget line which is tangent to $C$ lies below $A$. Naturally, this causes some unease with the Hicksian definition of real income because it suggests that although the consumer cannot consume her initial bundle any more, she still enjoys the same level of real income.

**The Slutsky analysis**

In the Slutsky analysis, we simply ask ourselves what the individual would choose if there were new relative prices but she is able to maintain her present consumption. This will reveal the net substitution effect, since real income (measured in terms of the ability to buy the initial bundle) remains unchanged. This idea is illustrated by the use of an imaginary budget line that goes through $A$ but reflects the new price ratio, as in Figure 2.21.

![Figure 2.21: The Slutsky analysis.](image-url)
As the new budget line, with changed relative prices, goes through A, it cannot be tangent to the indifference curve at A. Hence A can no longer be considered as the optimal choice (because the subjective rate of substitution in A is greater than the market exchange rate between X and Y). The individual will choose to be at point C, which is on a higher utility level.

To calculate the nominal equivalent to the real income effect we can now simply ask how much money would be needed to consume the bundle at A at the new prices. The answer here will be 75. This is because to consume the bundle at A at the new prices we need money amounting to

\[ P_0^X X_0 + P_0^Y Y_0 = 5 \times 5 + 10 \times 5 = 75 \]

Therefore, according to Slutsky’s definition of real income, the nominal equivalent to the income effect is only 100 – 75 = 25.

Think about these two different approaches. Will the substitution effect always be greater under Slutsky’s definition of real income than under Hicks’s?

**Normal and inferior goods**

Having distinguished between the income and substitution effects, we are now able to make a further distinction, that between normal and inferior goods. We will show that a normal good is defined as a good that has a positive income effect, while inferior goods have a negative income effect.

It is because we have provided an explanation in the form of the rational utility maximiser that we are able to distinguish:

a. between substitution and income effect, and

b. between inferior and normal goods.

If we had chosen to accept the downward sloping demand curve as a ‘law’ or axiom, rather than as the outcome of more fundamental processes, we would not have been in this position.

Bear in mind that the net-substitution effect is always inversely related to the change in the relative price (the market exchange rate between the goods): the cheaper (in relative terms) a good becomes, the more we consume of it. This, we established, is due to the nature of utility functions and their indifference curves. Hence, any proposed distinction between goods cannot depend on the net-substitution effect. It must, therefore, depend entirely on the income effect.

In Figure 2.22, the price of X has fallen from \( P_0^X \) to \( P_1^X \). Net-substitution suggests that the individual will move from point A to point C. This is always true, regardless of whether the good is inferior or normal.
The move from the broken line at C to the new budget line requires a parallel shift, which, as we saw, is equivalent to an increase in nominal income. This, therefore, is the income effect. The increase in real income means that the individual can now buy more of all goods. It would be perfectly rational for the individual to choose to move to a point B anywhere on the new budget line, where utility is higher.

There are now two main possibilities:

1. the individual moves to the right of point C (which means that as income increases, the individual will want to consume more of X).
   
   In this case we say that X is a normal good, whose consumption increases with income.

2. the individual will choose to be on the left of C (which means that as income increases, the individual will want to consume less of X).
   
   In this case, we say that X is an inferior good, in the sense that the consumption of X decreases as income increases.

Try to think of some examples of ‘inferior’ goods, and explain why consumption of them decreases with income.

The position of B, in the end, depends on where indifference curves are, and which option will be chosen is entirely a matter of personal preferences. Therefore, being an inferior or normal good is not an intrinsic characteristic of a good. It is the way in which individuals see them which makes us consider them as either normal or inferior.

We now have three possible effects that a change in the price of a good can have on the quantity of it which will be demanded. Figure 2.23 offers a summary:
Figure 2.23: Normal, inferior and Giffen goods and their demand schedules.

In the case of a normal good, net-substitution and the income effect seem to be working in the same direction (A to B'). In the case of an inferior good the income effect appears to work in the opposite direction to the net-substitution effect. There are now two possibilities. Either:

i. the net-substitution effect is greater (in absolute values) than the income effect (A to B'), or

ii. the net-substitution effect is smaller (in absolute values) than the income effect (A to B').

We distinguish the latter (ii) from the general group of inferior goods by naming it a **Giffen** good. For example, Sir Robert Giffen observed that an increase in the price of wheat led to an increase in the demand for bread by nineteenth-century peasants. However, it is widely believed by many that Giffen goods do not exist in practice as it is unlikely to be the case that a negative income effect would be strong enough to offset the substitution effect. As before, we can translate these price effects into a demand function, on the right-hand side. We see that the demand for a normal good will tend to be flatter than the demand for an inferior good. The demand for an inferior good for which the net-substitution effect is dominant continues to be downward sloping, while that of an inferior good for which the income effect dominates (a Giffen good) is upward sloping.

**Complements and gross substitutes**

Since we are defining our preferences over the entire space of economic goods, decisions we make about one good will influence our decisions about the other goods. Where there are two goods, choosing the quantity of $X$ also means choosing the quantity of $Y$. Therefore, the demands for the two goods are interrelated.

Consider again the fall in the price of $X$ as discussed above, but now, let us concentrate on what happens to the quantity of $Y$ demanded (**Figure 2.24**).
Figure 2.24: Complements and gross substitutes.

There are, in principle, two possibilities. Either:

i. $B$ (the new optimal point) falls above $A$, which means we **increase** the consumption of $Y$ when the price of $X$ falls, or

ii. $B$ falls below $A$, in which case we **reduce** the consumption of $Y$ as the price of $X$ falls.

Where the consumption of $Y$ increases when the price of $X$ falls (case i), we say that $X$ and $Y$ are **complements**. A fall in the price of $X$ suggests that people will buy more of $X$ (unless $X$ is a Giffen good). If they also consume more of $Y$, then, in a sense, the two goods ‘go together’, or are complementing each other. Common examples of such goods are cars and fuel. As the price of fuel falls, there will be greater use of private cars and greater consumption of fuel.

In a case where the consumption of $Y$ falls as the price of $X$ falls, we say that $X$ and $Y$ are **gross substitutes**. (We use ‘gross’ to distinguish it from the substitution which we discussed before). Again, a fall in the price of $X$ will lead to more consumption of $X$, unless it is a Giffen good.

If consuming more of $X$ means consuming less of $Y$, we must feel that we can **substitute** $X$ for $Y$. A typical example can be the use of private cars and public transport. When the price of public transport drops, people will tend their private car less and travel more on public transport.

**A generalised demand function**

Let us now summarise and generalise the derivation of demand that we have pursued so far:

1. The desirability of economic goods presents itself in the form of preferring more to less; a rational individual has **consistent** preferences over the world of economic goods which can be represented by a real number function called the **utility function**.

2. This means that individuals’ demand for all goods is determined simultaneously.
3. The individual confronts scarcity in the form of a **budget constraint**.

4. A utility maximising individual will choose the bundle where their **willingness to pay** equals the **market price** or exchange rate between the goods.

![Figure 2.25: Deriving demand.](image)

The choice of both \( X_0 \) and \( Y_0 \) in **Figure 2.25** reflects the individual’s demand for \( X \) and \( Y \).

5. The demand for \( X \), therefore, depends on those parameters that determine the position of point \( A \).

6. Point \( A \) is determined by the **utility function** (which determines the shape and position of the indifference curve), and the position of the budget line.

7. The position of the budget line is determined by Income (\( I \)) and the prices \( P_X \) and \( P_Y \).

8. **Hence, demand is a function of the utility function, income, and prices.** We can write it as a function:

\[
X' = D(P_X, P_Y, I, U)
\]

and for given tastes (assuming no change in \( U \)):

\[
X' = D(P_X, P_Y, I)
\]

(Note that \( D \) is the demand function and \( X' \) is the quantity demanded).

9. The first and immediate property of this demand function is that it is **homogeneous of degree 0**. This means that if we, say, doubled all variables \( (P_X, P_Y, I) \), the choice of \( X \) will remain unchanged, since the position of the budget line is unaffected by such a change. (The budget line is determined by the intercepts: \( I/P_X, I/P_Y \) and the slope: \( P_Y/P_X \)).

10. The quantity demanded of \( X \) is inversely related to the price of \( X \) if \( X \) is a normal or non-Giffen inferior good.
11. The quantity demanded of $X$ is directly (positively) related to the price of $Y$ if $X$ and $Y$ are **gross substitutes** and inversely related if $X$ and $Y$ are **complements**.

12. The analysis can be generalised in the following way: if there are $n$ goods in the economy, denoted by $(X_1, \ldots, X_n)$, the demand for good 1 will take the following form:

   $$X_1^d = D(P_1, \ldots, P_n, I)$$

   This $D$ is homogeneous of degree 0 as well: everything we said about $P_X$ holds here for $P_1$; and everything we said about $P_Y$ is true here for any of the other prices.

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**Market demand**

We have now derived the individual demand schedule through our analysis of the rational utility maximiser, and found, as expected, an inverse relationship between quantity demanded and price. **Market demand** is simply the total quantity demanded. It is the sum of the quantities demanded by each individual when their willingness to pay equals the market price. The right-hand diagram is thus a summary of the three diagrams to its left.

Technically, this is called **horizontal summation**. **Figure 2.26** is a geometrical presentation of this idea. It shows three individuals (which could also be groups of individuals) and a total market demand (on the right).

**Figure 2.26: Market demand.**

In this figure, $X_1^0$, $X_2^0$, and $X_3^0$ are the quantities demanded by each individual at the price $P_0$, and $X_T^0 = X_1^0 + X_2^0 + X_3^0$ is the total quantity demanded in the market.

**Demand price elasticity**

**Reading**

- **LC** Chapter 4 pp.38–44.
- **BFD** Chapter 4 pp.65–84.

Consider the demand schedule shown in **Figure 2.27**, which represents the relationship between the quantity of $X$ demanded and its price (we assume all other prices and income are fixed). For the purpose of our analysis, we can treat this either as the market demand, in which case income is that of the entire population, or as the demand of a (representative) individual.
Figure 2.27: Deriving demand price elasticity.

If the current price is \( P_0 \) and the quantity demanded at that price is \( X_0 \), then total spending on \( X \) is \( P_0 X_0 \). This is both the consumers' expenditure and the producing firms' revenue.

If the price of \( X \) falls to \( P_1 \) and the quantity demanded increases to \( X_1 \), total consumer spending will now be \( P_1 X_1 \).

Geometrically, we can say that the total spending at point \( A \) is:

\[ \alpha + \beta \]

and at point \( B \) the total spending is:

\[ \beta + \gamma \]

The question we wish to investigate is what will happen to consumer spending (or firms' revenue) if the price of \( X \) falls. What factors influence whether spending (revenue) changes in direct or inverse relation to the change in price?

We begin by investigating the case when revenues (and spending) decrease as the price decreases. Revenue at \( A \) will be greater than revenue at \( B \). Given our previous notation, this means that \( \alpha + \beta > \beta + \gamma \).

Since \( \beta \) is a common area, we need \( \alpha > \gamma \) for this direct relationship to hold. But when is this the case? What exactly are these areas?

If the quantity demanded had stayed at \( X_0 \) after the price has fallen from \( P_0 \) to \( P_1 \), the loss in revenues (or, from the point of view of the consumer, the savings on purchases) would be \( \alpha \). Therefore, we can write \( \alpha = dP X \).

Similarly, \( \gamma \) represents the gains on the new sales (or the extra spending on the added consumption). In other words, \( \gamma = dX P \).

Hence, the inequality \( \alpha > \gamma \) holds if:

\[ dP X > dX P \]

Note that both \( \alpha \) and \( \gamma \) are positive numbers. However, when \( dP > 0, dX < 0 \). This is because the demand curve is downward sloping generally.

Hence, we are really looking at the absolute values of the changes. Writing the equation in terms of absolute values and rearranging it, we find:

\[ |dP X| > |dX P| \]

Dividing through by \( dP X \) we get

\[ \frac{1}{|dP X|} > \frac{|dX P|}{dP X} = \frac{|dX / X|}{|dP / P|} = |\eta| \]

\( |\eta| \) denotes absolute values. Normally, when \( dP < 0, dX > 0 \), but geometrically we do not have negative areas.
We see that $\alpha > \gamma$ whenever $\eta$ is less than 1. This $\eta$ is called the **price elasticity of demand**, and is defined as the proportional change in quantity over the proportional change in price. Being less than unity means that the proportional change in price (in absolute values) is greater than the proportional change in quantity, and revenues (or consumer spending) will change in direct relation to the change in price. **Figure 2.28** depicts such conditions. It is easy to see that $\alpha > \gamma$ (or $|\eta| < 1$).

**Figure 2.28:** Inelastic demand.

This means that a reduction in price will reduce consumers’ spending (and firms'revenues) on that good.

Similarly, **Figure 2.29** depicts a typical case where $\alpha < \gamma$ ($|\eta| > 1$).

**Figure 2.29:** Elastic demand.

We can see from these two graphs that when $|\eta| < 1$, the demand curve is quite steep, while when $|\eta| > 1$, it is quite flat. Another way of describing this situation is that in **Figure 2.28**, the quantity demanded changes relatively little for a given change in price (this is **inelastic demand**), while in **Figure 2.29** it changes a lot (this is **elastic demand**).
Example 4

Let us now return to the bridge problem posed at the beginning of the chapter. To remind you, what we had there was a government having to decide on whether to build a bridge in a case where demand and supply do not intersect. Market research has produced the demand schedule shown in Figure 2.30, and the engineering investigation produced a bridge of minimum capacity of $T$ crossings per day at a cost of $C$.

![Figure 2.30: A bridge revisited.](image)

Now that we have derived demand from utility we know that when an individual chooses a quantity at a given price they are in a position where \( \frac{MU_x}{MU_y} = \frac{P_x}{P_y} \) units of $Y$ per $X$.

Suppose that crossing the bridge is $X$. So when an individual answers the questionnaire by saying that they would cross 5 times a day if the price was £10, it means that for a given price of other goods $Y$ (say, £5):

\[
\frac{MU_x}{MU_y} = \frac{P_x}{P_y} = 2 \text{ units of } Y \text{ per } X
\]

Put another way, at 5 crossings a day, the marginal utility of crossing (measured in terms of $Y$ which the individual is willing to give up) equals 2 units of $Y$ per crossing. As the price of $Y$ is £5, in money terms this means that bridge users are willing to pay (2 units of $Y$ times £5 per unit =) £10. These £10 denote the money value of the marginal utility from crossings at the point where the individual crosses 5 times. If, then, the demand schedule denotes a money value for the marginal utility of each crossing, adding up all these marginal utilities (vertical lines from the $X$-axis to the demand schedule) will give us the individual’s total utility at a given price of $Y$.

In other words, the area underneath the demand schedule gives us a money value for the utility of a rational individual. Thus the total amount of benefits generated by the bridge is the area trapped in the demand triangle in Figure 2.31.

As we have identified the overall benefit $B$ in money terms, the government can examine whether this exceeds the overall cost ($C$) and thus decide whether or not to build the bridge.

Of course, in reality the problem is more complex, as there are issues of financing to be considered. Still, what I hope you were able to see is how the use of utility functions helped us to analyse and evaluate the demand schedule. This analysis enabled us to create a framework in which we can...
gain further insights into the nature of demand. In the process, we found that even when supply and demand do not intersect (as in the bridge example), our interpretation of the area under the demand schedule, or the money value of utility, enabled us to make an informed statement about the desirability of building the bridge.

**Self-assessment**

**Check your knowledge**

Check back through the text if you are not sure about any of these.

- Define the concepts of utility, equilibrium price, transitivity, marginal utility, indifference points and indifference curves, income effect, substitution effect, 'inferior' and 'normal' good completeness and gross substitutes, price elasticity, and real income.
- Derive utility and indifference curves.
- Use utility and demand curves to analyse problems involving choice, utility maximisation, substitution and income effects, and price elasticity of demand.

Give an example of:

- a case where changes in taste or fashion lead to an increase in both supply and price
- the formula for expressing preferences between three different goods
- a Giffen good.

**Test your understanding**

In this section, you will find a set of problems of the kind you will meet in the exam. The answers follow on page 84.

Try to answer the questions without looking at the answers. After you have answered all the questions, compare your answers with someone else who is studying this course. If there is no other student you can consult, choose a (patient) friend or family member and try to explain to them the issues involved. It doesn’t matter if they don’t know anything about economics: this will force you to explain the subject in a way that will help you
understand things which you would not have understood otherwise. Only after all these trials should you compare your answers with the answers in the book.

Question 1
When the price of \( X \) is 3 and the price of \( Y \) is 3, an individual consumes a bundle of \( X = 4, \ Y = 4 \). When the price of \( X \) has become 1 and the price of \( Y \) 5, the individual chooses a bundle of \( X = 3, \ Y = 5 \). Therefore, the consumer prefers \( (3, 5) \) over \( (4, 4) \). True or false? Explain.

Question 2
In a world of two goods, when the demand elasticity of good \( X \) is greater than unity, \( X \) and \( Y \) must be gross substitutes and \( X \) is more likely to be a normal good. True or false? Explain.

Question 3
A good is a normal good whenever the substitution and income effects work in the same direction. True or false? Explain.

Question 4
The Slutsky substitution effect is always greater than the Hicksian substitution effect. True or false? Explain.

Question 5
A company considers a package to help employees with the running cost of their cars. It considers two options:
A. to offer a fixed amount of money towards the use of the car in addition to a cost-free usage for the first \( X_0 \) miles;
B. to participate in the actual cost of running the car (i.e. pay a certain amount, \( a_o \), per mile used).

a. Let \( X \) represent mileage of car usage and \( Y \) all other goods. Draw each of the options while analysing the individual's response to the proposed change (i.e. discuss the income and substitution effects);
b. which of the two options will the employee prefer if the company decided to spend the same amount of money under the two options?
c. will your answer to (2) change had option A included only free mileage?
d. which of the two options would the company prefer if it aims at achieving the same real income improvement at a lower cost?

Question 6
A telephone company charges its customers a fixed sum of \( T \) for the first \( X \) calls they make in a given period. Every extra call is then charged at the price of \( P \) a call. The company would like to replace the existing arrangement with a new one. It considers two alternatives:
A. abolish the fixed payment and charge a lower price for each call;
B. increase the number of calls allowed under the same fixed payment and increase the price of every extra call.

Assume that customers always make more calls than are covered by the fixed payment.
a. Draw the budget constraint confronting customers under the initial scheme;
b. Draw option (A) and consider whether customers are likely to be better
or worse off. Can the company choose a price where customers are equally well off as under the original scheme? What will happen to the number of calls in such a case?

c. Draw option (B) and consider whether customers are likely to be better or worse off. Had the option been designed in such a way as to allow individuals to consume the number of calls they would be able to consume under (A), will it be a better or worse option for the consumer?

d. If you knew that most customers use the phone only slightly above what is covered by the fixed payment, which of the schemes would you recommend? How would you advise the company if this was not the case?

**Question 7**

It is better to give the poor a subsidy for food rather than an income supplement which they are likely to spend on other goods and alcohol.

Suppose individuals consume only two goods, $X$, which is food and $Y$, which is other goods (including alcohol), and that they have an income of $I$.

a. Show the effects on consumption of paying a subsidy of $s$ per unit of $X$ consumed;

b. Show the effects on consumption of paying income supplement of $S$;

c. Compare the effects of the two schemes assuming that government spending on each individual is the same in both cases (this means that if under the subsidy scheme the individual chooses $X_s$ then $sX_s = S$);

d. Comment on the statement.

**Answers**

**Question 1**

This is a question about choice. It could be analysed by the use of ‘revealed preference approach’ (for those students who are familiar with it) or by simple utility analysis. Of course it is the latter which we expected to find:

![Figure 2.32](image-url)
We have the following situation: At point $A$, $(4, 4)$, in Figure 2.32, if the consumer is rational his choice will exhaust the following budget line:

$$P_0 X_0 + P_0 Y_0 = 3 \times 4 + 3 \times 4 = 24$$

At $B$ his income is obviously greater:

$$P_1 X_1 + P_1 Y_1 = 1 \times 3 + 5 \times 5 = 28$$

However, he could have afforded point $B$ on the initial budget line:

$$P_0 X_1 + P_0 Y_1 = 3 \times 3 + 3 \times 5 = 24$$

which suggests that the individual chose $A$ when $B$ was available. So, if anything, the individual prefers $A$ over $B$. It is easy to see, using indifference curve analysis, that the individual behaves irrationally by choosing point $B$.

**Question 2**

In order to analyse the nature of $X$ and its relationship with $Y$ we must investigate a change in the price of $X$. Suppose that the price of $X$ fell. This leads to the following diagram:

![Diagram](image)

**Figure 2.33**

Using the information that the demand elasticity for $X$ is greater than unity, we can conclude that as a result of the fall in the price of $X$, spending on $X$ will rise. As nominal income is unchanged, spending on $Y$ must come down.

As the price of $Y$ too, remains unchanged, the quantity demanded of $Y$ must fall. This suggests that $X$ and $Y$ are gross substitutes. In the above diagram, points on the new budget constraint where the consumption of $Y$ has decreased are indicated by the heavy line. We can see that such points are likely to lie to the right of $C$, therefore $X$ is more likely to be a normal good.
**Question 3**
False. A simple counter example like the case where income is given in kind (as below) should be sufficient:

![Figure 2.34](image)

The individual gets income in kind: \( I_K = (X_K, Y_K) \). At A he sells some of his endowment in \( X \) and buys some more \( Y \). When the price of \( X \) falls, the new budget line will have to go through his point of income (because he can always choose not to trade). Substitution considerations will lead him to C while the fall in real income will mean that the good is normal only if the income and substitution effects work in the opposite directions.

**Question 4**
False. There are three components to this question:

a. The difference between Hicks and Slutsky definitions of ‘real income’.

b. Analysing the fall in the price of \( X \) and showing that the Slutsky substitution effect is greater when utility functions are homothetic and the good is normal (the left-hand side diagram below).

c. Analysing the fall in the price of \( X \) and showing that in the case of an inferior good, the Hicksian substitution effect is greater.

**Note:** the reverse will be true if you analyse an increase in the price of \( X \).
Question 5

This question combines both an analysis of the budget line and the theory of consumer choice. In the latter part, the main analytic elements are income and substitution effects.

a. An employee is offered by a company a package to help in the running cost of his company car. There are two options:

1. A lump-sum payment \( L \) towards the use of the car as well as a certain amount of free usage (measured in miles). This option is captured in the left-hand part of Figure 2.36.

2. a ‘subsidy’ per mile used. This option is captured in the right-hand diagram of Figure 2.36.

Figure 2.36

In both cases the individual will increase the use of the car. In the case of the ‘lump-sum’ payment, there will be no substitution effect, since the relative price doesn’t change, while in the case of the ‘subsidy’ there will be both income and substitution effects.

b. To spend the same on the two schemes means that the amount of money paid out to the individual according to their use of the car should be the same as the money paid to them when the payment is independent of that use. This means: \( a_0X_0 = L \).
In other words, the budget line under offer (1) must cross the budget line under offer (2) at the point where the individual would choose to be had he received (2). We can show this formally: at $B$

(option 2) \[ (P^0_x - a_0)X_0 + P^0_y Y = I_0 \Rightarrow P^0_x X_0 + P^0_y Y = I_0 + a_0 X_0 \]

(option 1) \[ P^0_x X_0 + P^0_y Y = I_0 + L \quad a_0 X_0 = L \]

If the two offers are to be of equal money value, point $B$ must be on both budget lines.

It is evident that the individual will prefer option 1, since this represents a higher level of utility.

c. The answer will not change, but the company will need to offer more free miles.

d. If the company wants to achieve a certain real income improvement, say $U_1$, then it is easy to see that the cheapest option will be option 1:
As the budget line of option 1 is tangent to the indifference curve upon which the choice of option 2 has been made, the choice under option 2 (point B) will not be feasible had option 1 been finally offered. Therefore, the money which the company will spend under option 1 to achieve the same utility as under option 2 will be much reduced.

**Question 6**

This question has two major components. One is the budget constraint and its possible shapes; the other, a **comparative analysis of individual’s choice**. It aims at showing how economists may use abstract frameworks to provide practical recommendations.

The pretext is the pricing policy of a telephone company. As no information is provided regarding differences in costs or the market structure, it implies that the criterion for choosing a scheme is a different one. From reading the question in its entirety, we can deduce that this is a firm which is more concerned with its public image than with its position in the market.

This is clearly an indifference curves analysis, simply because the main question here is whether or not the customer (a representative individual) will be better or worse off. The next step, therefore, is to translate the question into the language of the model.

Here, the real ‘jump’ is the transformation of the three pricing policies (the existing one, (A) and (B)) into forms of budget constraints.

The initial budget constraint is drawn in **Figure 2.39**.

![Figure 2.39](image)

**Figure 2.39**

e. **Figure 2.40** details the diagrams that should emerge. Scheme (A) is drawn in the left diagram. Note that if consumers were initially at P they **might** be worse off. Had they been initially at T they will definitely be better off. In the right-hand diagram, it is shown how a price can be set such that their utility remained unchanged (this is an example of how at a point like P, individuals will **not** be made worse off).
f. The general principle is depicted in the left diagram of Figure 2.41. At point $P$, they will definitely be better off. Had they been initially at point $T$ they might be worse off. This is exactly the opposite of the previous scheme. In the diagram on the right we can see the circumstances where scheme (B) is designed in such a way as to ensure the feasibility of the choice under scheme (A). It is clear from this that in such a case, individuals would rather have scheme (B).

Figure 2.41

g. This requires a more general answer: had the consumers been initially at point $P$ on Figure 2.42, scheme (B) is likely to appeal to them more. Had they been initially at point $T$, scheme (A) would be more appealing. One must bear in mind that we have no information about the costs of the two schemes. Assuming that they cost the same, the company would want to appear as having the consumers’ benefit in mind.
Chapter 2: Individual choice

Figure 2.42

Note: this question is a good example how one can conduct a rigorous discussion even when there is no single answer. This, to a great extent, is what economics is all about.

Question 7

As in Question 6, this question has the same major components: the budget constraint and the comparative analysis of individual’s choice. The pretext here is the famous problem of subsidising goods or individuals. Here, the analysis is conducted from the point of view of the affected individuals. Other social issues and the difference in administration costs are neglected, since there is a complete lack of any information regarding the cost side of the two schemes.

Just like in the previous question, the analytical framework is clearly an indifference curves analysis, simply because the main question here is whether or not are presentative individual will be better or worse off. The next step, therefore, is to translate the question into the language of the model through the transformation of the two policy tools into forms of budget constraints.

a. and b. The diagram on the left of Figure 2.43 depicts the effects of a subsidy (s) on the budget line and the possible consumption of X. The diagram on the right depicts the effects of an income supplement S on the budget line.

Figure 2.43
c. Here, the main test lies in interpreting the equal spending (i.e. \( sX_s = S \)) and the relative positions of the two budget lines:

![Diagram showing budget lines and indifference curves]

**Figure 2.44**

Notice that \( sX_s = S \) means that the income-supplement budget line will always go through whichever choice the individual would have made under the subsidy scheme.

d. Using **Figure 2.44**, we can use indifference curves analysis to show that the income supplement will be preferred by individuals. Note that the indifference curves which are tangent to the two budget constraints will not be the same!