This is an extract from a subject guide for an undergraduate course offered as part of the University of London International Programmes in Economics, Management, Finance and the Social Sciences. Materials for these programmes are developed by academics at the London School of Economics and Political Science (LSE).

For more information, see: www.londoninternational.ac.uk
# Contents

**Introduction** ........................................................................................................ 1
  Terminology ................................................................. 1
  Aims ........................................................................... 2
  Learning outcomes ...................................................... 3
  Syllabus ...................................................................... 3
  Reading advice ............................................................ 4
  Online study resources ................................................ 4
  Recommended study time ............................................ 5
  How to use the resources for this course ....................... 6
  Examination advice ..................................................... 7
  Using software for the course ....................................... 7
  Excel spreadsheets for the course ................................. 8
  Concluding remarks .................................................... 8
  List of abbreviations used in this subject guide .......... 9

**Chapter 1: Methodology** ................................................................. 11
  Essential reading ........................................................ 11
  Aims of the chapter ...................................................... 11
  Learning outcomes ...................................................... 11
  Introduction .................................................................. 11
  Evolution ..................................................................... 11
  Two Mines company .................................................. 13
  Discussion .................................................................. 16
  Philosophy .................................................................. 17
  Certainty versus uncertainty ....................................... 17
  Phases of an OR project ............................................... 18
  Methodological issues ............................................... 20
  Benefits ....................................................................... 25
  Links to other chapters ............................................... 26
  Case studies ............................................................... 26
  A reminder of your learning outcomes ......................... 27
  Sample examination questions .................................... 27

**Chapter 2: Problem structuring and problem structuring methods** .......... 29
  Essential reading ........................................................ 29
  Aims of the chapter ...................................................... 29
  Learning outcomes ...................................................... 29
  Introduction .................................................................. 29
  Problem structuring methods ...................................... 30
  Strategic options development and analysis (SODA) and
  JOURNEY (Jbrtly Understanding, Reflecting, and NEgotiating strategy) Making ....... 32
  Soft systems methodology (SSM) .................................. 35
  Strategic choice (SC) ................................................... 40
  Choosing and applying PSMs ....................................... 45
  Education ................................................................... 45
  Links to other chapters ............................................... 45
  Case studies ............................................................... 46
Supply chain management (SCM) ................................................................. 108
Links to other chapters ........................................................................... 109
Case studies .............................................................................................. 109
A reminder of your learning outcomes .................................................... 109
Sample examination questions ................................................................ 109
Table for the standard Normal distribution .............................................. 110

Chapter 6: Markov processes ................................................................. 111
Essential reading ...................................................................................... 111
Spreadsheet ............................................................................................ 111
Aims of the chapter .................................................................................. 111
Learning outcomes ................................................................................... 111
Introduction ............................................................................................. 111
Solution procedure .................................................................................. 112
Long-run .................................................................................................... 116
Three states .............................................................................................. 117
Estimating the transition matrix ............................................................. 121
Comment ................................................................................................. 123
Applications ............................................................................................ 124
Links to other chapters ............................................................................ 124
Case studies ............................................................................................. 124
A reminder of your learning outcomes .................................................... 124
Sample examination questions ................................................................ 124

Chapter 7: Mathematical programming – formulation ......................... 125
Essential reading ...................................................................................... 125
Aims of the chapter .................................................................................. 125
Learning outcomes ................................................................................... 125
Introduction ............................................................................................. 125
Overview .................................................................................................. 125
Blending problem ..................................................................................... 126
Production planning problem ................................................................. 127
Factory planning problem ........................................................................ 129
Integer programming .............................................................................. 131
Links to other chapters ............................................................................ 134
Case studies ............................................................................................. 134
A reminder of your learning outcomes .................................................... 135
Sample examination questions ................................................................ 135

Chapter 8: Linear programming – solutions ........................................... 137
Essential reading ...................................................................................... 137
Spreadsheet ............................................................................................ 137
Aims of the chapter .................................................................................. 137
Learning outcomes ................................................................................... 137
Introduction ............................................................................................. 137
Graphical solution for two variable LPs ................................................. 138
Simplex ..................................................................................................... 140
Production planning problem ................................................................. 140
Excel solution .......................................................................................... 141
Problem sensitivity ................................................................................... 144
Mathematical programming – further considerations ............................. 150
Links to other chapters ............................................................................ 153
Case studies ............................................................................................. 153
Simple M/M/1 example................................................................. 199
Simulation..................................................................................... 202
Links to other chapters ............................................................... 208
Case studies.................................................................................. 208
A reminder of your learning outcomes........................................ 209
Sample examination questions.................................................. 209

Appendix 1: Sample examination paper ................................. 211
Appendix 2: Sample Examiners’ commentary ....................... 219

Important note ........................................................................... 219
Comments on specific questions .............................................. 219
Introduction

Welcome to the subject guide for **MN3032 Management science methods**. We hope that you enjoy the subject and benefit from it, not just in terms of a good performance in your examination, but also in terms of knowledge and analytical skills that you can use at some point in your future career.

Management, in the modern world, is becoming increasingly complex. Problems are becoming more difficult to solve and the timescales available to solve them are becoming shorter. For many problems, data (numbers) are available and these data need to be properly considered and analysed to help in solving such problems.

While, for the gifted few, problem-solving may be an easy and natural process, we believe that the majority of managers benefit from a structured approach to problem-solving.

Management science can be defined as the application of scientific and systematic procedures, techniques and tools to operational, strategic and policy problems in order to help develop and evaluate solutions to problems encountered within management. Management science includes all rational approaches to management decision-making that are based on an application of scientific and systematic methodologies.

Both management science as a discipline, and management scientists as individuals with specialised training, aid management by providing a structure to decision-making situations whose complexity, and/or level of uncertainty, makes intuition an unsafe guide. The distinctive feature of the management science approach is the construction of an explicit, simplified model of relevant aspects of the situation under study. Such models are often based on mathematical or statistical formulations, but may at other times have a more qualitative character.

There are a number of areas where specific structured techniques to aid in decision-making have been developed. Such techniques have proved their worth historically and have made a quantitative difference to the way in which organisations work and to the quality of their decision-making. You will be introduced to some of these techniques in this subject guide.

Above all, what you need to gain from this subject guide is the knowledge that there are techniques available to help you solve management problems in a **structured** and **logical** way.

**Terminology**

Management science can also be described by the alternative name of operational research (OR). Some of you may have met the terms management science (MS), operational/operations research (OR), and OR/MS before. Often some, or all, of these terms are used interchangeably. In this subject guide we use the term OR throughout.

**Analytics** (or data analytics) is another term that deals with the same issues as we consider in OR – we have data; we have decisions to be made; how can we analyse the data to help us make an appropriate decision?
Analytics is often linked to 'big data'. Modern organisations are accumulating, perhaps minute by minute, increasing amounts of data in their interactions with their customers. The idea is that if (somehow) these data could be analysed, the organisation would be able to make better decisions, for example with regard to its product/service offerings to customers.

Here the volume of data (so ‘big data’) preclude any analysis by a single individual or team of people. To take just one example, if you were to be personally presented with information on the interactions that Facebook has with its billions of users in just a single day do you think that you could sensibly analyse it? Of course not, the sheer volume of such information would be over-whelming. Hence structured approaches to analysing data are needed.

The Institute for Operations Research and the Management Sciences (INFORMS, www.informs.org), a USA-based academic/professional society that deals with analytics, identifies the following approaches:

- **descriptive analytics**
  - preparing and analysing historical data
  - identifying patterns from samples for reporting of trends

- **predictive analytics**
  - predicting future probabilities and trends
  - finding relationships in data that may not be readily apparent with descriptive analysis

- **prescriptive analytics**
  - evaluating and determining new ways to operate
  - targeting business objectives
  - balancing all constraints.

A number of the topics you encounter in this subject guide fall under the analytics banner. One thing to note, however, is that this subject guide exists in the educational world, not the real world. In the educational world, by its very nature, any examples considered need to be ‘small data’ examples.

However, although the quantitative/analytic techniques and tools you see below will be illustrated on small data many of them are capable of scaling to deal with big data.

### Aims

The aims and objectives of this course are to:

- enable you to see that many managerial decision-making situations can be addressed using standard techniques and problem structuring methods
- provide a comprehensive and concise introduction to the key techniques and problem structuring methods used within management science that are directly relevant to the managerial context
- enable you to see both the benefits, and limitations, of the techniques and problem structuring methods presented.
Learning outcomes

On completion of the course, you should be able to:

• discuss the main techniques and problem structuring methods used within management science
• critically appraise the strengths and limitations of these techniques and problem structuring methods
• carry out simple exercises using such techniques and problem structuring methods themselves (or explain how they should be done)
• commission more advanced exercises.

Syllabus

The topics dealt with in this course (in chapter order) are:

**Problem structuring and problem structuring methods:** problem structuring methods such as JOURNEY (JOintly Understanding, Reflecting, and NEgotiating strategY) making, Soft Systems Methodology and Strategic Choice.

**Network analysis:** planning and control of projects via the critical path; float (slack) times, cost/time trade-off, uncertain activity completion times and resource considerations.

**Decision making under uncertainty:** approaches to decision problems where chance (probability) plays a key role; pay-off tables; decision trees; utilities and expected value of perfect information.

**Inventory control:** problems that arise in the management of inventory (stock); Economic Order Quantity, Economic Batch Quantity, quantity discounts, probabilistic demand, Materials Requirements Planning, Just-in-Time, Optimised Production Technology and supply chain issues.

**Markov processes:** approaches used in modelling situations that evolve in a stochastic (probabilistic) fashion though time; systems involving both non-absorbing and absorbing states.

**Mathematical programming formulation:** the representation of decision problems using linear models with a single objective which is to be optimised; the formulation of both linear programs and integer programs.

**Linear programming solutions:** the solution of linear programs; the numeric solution of two variable linear programs, sensitivity analysis and robustness.

**Data envelopment analysis:** assessing the relative efficiency of decision-making units in organisations; input/output definitions, basic efficiency calculations, reference sets, target setting and value judgements.

**Multicriteria decision making:** approaches to decision problems that involve multiple objectives; analytic hierarchy process which considers the problem of making a choice, in the presence of complete information, from a finite set of discrete alternatives; goal programming which considers, via linear programming, multicriteria decision problems where the constraints are ‘soft’.

**Queueing theory and simulation:** the representation and analysis of complex stochastic systems where queueing is a common occurrence; M/M/1 queue; discrete event simulation.
Reading advice

Essential reading

There are two texts associated with the readings given at various points throughout this subject guide. These are:


Both of these books are recommended for purchase/reference and will be listed as 'Anderson' and 'Rosenhead' respectively throughout this guide.

Anderson is available to purchase as separate chapters from:

http://edu.cengage.co.uk/catalogue/product.aspx?isbn=1408089401

Detailed reading references in this subject guide refer to the editions of the set textbooks listed above. New editions of one or more of these textbooks may have been published by the time you study this course. You can use a more recent edition of any of the books; use the detailed chapter and section headings and the index to identify relevant readings. Also check the VLE regularly for updated guidance on readings.

Case studies

In each chapter we have listed a number of case studies. You will see that these are often quite short. The idea here is that we expose you to a number of different practical situations where the tools and techniques which you studied have been applied. We would encourage you to read these case studies to see the range of areas to which the topics presented in this subject guide have been utilised.

Online study resources

In addition to the subject guide and the Essential reading, it is crucial that you take advantage of the study resources that are available online for this course, including the virtual learning environment (VLE) and the Online Library.

You can access the VLE, the Online Library and your University of London email account via the Student Portal at:

http://my.londoninternational.ac.uk

You should have received your login details for the Student Portal with your official offer, which was emailed to the address that you gave on your application form. You have probably already logged in to the Student Portal in order to register. As soon as you registered, you will automatically have been granted access to the VLE, Online Library and your fully functional University of London email account.

If you have forgotten these login details, please click on the 'Forgotten your password' link on the login page.

The VLE

The VLE, which complements this subject guide, has been designed to enhance your learning experience, providing additional support and a
sense of community. It forms an important part of your study experience with the University of London and you should access it regularly.

The VLE provides a range of resources for EMFSS courses:

- Self-testing activities: Doing these allows you to test your own understanding of subject material.
- Electronic study materials: The printed materials that you receive from the University of London are available to download, including updated reading lists and references.
- Past examination papers and Examiners’ commentaries: These provide advice on how each examination question might best be answered.
- A student discussion forum: This is an open space for you to discuss interests and experiences, seek support from your peers, work collaboratively to solve problems and discuss subject material.
- Videos: There are recorded academic introductions to the subject, interviews and debates and, for some courses, audio-visual tutorials and conclusions.
- Recorded lectures: For some courses, where appropriate, the sessions from previous years’ Study Weekends have been recorded and made available.
- Study skills: Expert advice on preparing for examinations and developing your digital literacy skills.
- Feedback forms.

Some of these resources are available for certain courses only, but we are expanding our provision all the time and you should check the VLE regularly for updates.

**Making use of the Online Library**

The Online Library contains a huge array of journal articles and other resources to help you read widely and extensively.

To access the majority of resources via the Online Library you will either need to use your University of London Student Portal login details, or you will be required to register and use an Athens login: http://tinyurl.com/ollathens

The easiest way to locate relevant content and journal articles in the Online Library is to use the **Summon** search engine.

If you are having trouble finding an article listed in a reading list, try removing any punctuation from the title, such as single quotation marks, question marks and colons.

For further advice, please see the online help pages: www.external.shl.lon.ac.uk/summon/about.php

**Recommended study time**

With regard to the time that you will need to spend studying this subject, it is obviously difficult to be precise since this will naturally vary from student to student. Were the material in this subject guide to be presented as a university course with you sitting in lectures then as a rough indication each chapter might require four hours of lectures and two hours of tutorial time in order to get over the basic concepts. This is equivalent to a total of six contact hours in the classroom. We would expect International Programmes students to spend two to three times as long working independently in order to learn the material and attempting
sample examination questions. This implies a total of 18 to 24 hours of work per chapter. Any time spent in final revision for examination assessment would not be included in this total.

How to use the resources for this course

Using the guide

This subject guide is designed to:

- introduce you to the material you are expected to learn
- direct you towards appropriate further reading.

It is not designed to be a self-contained learning text. In other words, you will need to consult the essential readings to learn the material presented in this subject guide. In addition, those readings contain many more worked examples, as well as self-test problems, than are presented here. Study of those is expected in order for you to master the material presented in this subject guide.

This subject guide is intended to be used in such a way that you progress through it from Chapter 1 to Chapter 11 in order. Some of you may wish to study the subjects within this guide in a different order. This is perfectly acceptable but please note the following:

- Chapter 1 must be studied first
- Chapter 7 must be studied before Chapter 8
- Chapters 7 and 8 must be studied before Chapter 9
- Chapters 7 and 8 must be studied before Chapter 10.

In terms of subject guide content, the chapters can be grouped as follows:

- **introduction**, Chapter 1, this chapter sets the scene for the subject guide
- **problem structuring**, Chapter 2, this chapter introduces methods that can be used when the problem to be faced is essentially qualitative in nature and when a more quantitative (analytic) approach would be inappropriate
- **mathematical programming**, Chapters 7, 8 and 10 (as it relates to goal programming), these chapters cover topics related to formulating a decision problem in mathematics and its subsequent numeric solution (typically in this subject guide via Excel). Here we only consider deterministic topics, so chance (probability) plays no role
- **decision analysis**, here a split can be made between deterministic topics (Chapters 3, 5 and 9) and stochastic topics (Chapters 4, 6 and 11), where chance (probability) plays a key role.

Using the readings

To help you with the readings, we have stated:

- all the reading associated with that chapter at the start of the chapter
- the relevant reading for a particular topic at various points within the chapter.

Working through the activities

Throughout the guide you will find a number of questions designed to provoke thought/action, typically either encouraging reflection on the
Introduction

topics raised and their applicability in real life, or asking for a calculation to check that you can correctly apply a technique. These are labelled ‘Activity’ throughout the text. **We would strongly encourage you to try these activities for yourself.**

---

**Examination advice**

**Important:** the information and advice given here are based on the examination structure used at the time this guide was written. Please note that subject guides may be used for several years. Because of this we strongly advise you to always check both the current Regulations for relevant information about the examination, and the VLE where you should be advised of any forthcoming changes. You should also carefully check the rubric/instructions on the paper you actually sit and follow those instructions.

The examination is three hours long. You will have to answer four questions (all carrying equal marks) from a choice of eight questions.

It should be noted here that you will be expected to do all the calculations in the examination by hand and that no computers or software (such as Excel) will be available to assist you. However, you are permitted to take an appropriate (basic, non-programmable) calculator into the examination for this subject. **This calculator must comply in all respects with the specification given in the Regulations.**

Remember, it is important to check the VLE for:

- up-to-date information on examination and assessment arrangements for this course
- where available, past examination papers and Examiners' commentaries for the course which give advice on how each question might best be answered.

**Sample examination paper**

The Sample examination paper (Appendix 1) should only be attempted once you have finished the subject guide and the recommended reading. An accompanying Examiners' commentary is also supplied.

**Using software for the course**

It is common nowadays for management science methods to be taught in conjunction with a simple (and inexpensive) computer package, typically for the personal computer (PC). This enables you to solve problems numerically yourself and to explore the effect of changing numbers, assumptions, etc. There are a number of such software packages, designed for the student learning environment, which are readily available.

In this subject guide we shall make use of examples solved using Excel, the Microsoft spreadsheet package. We chose this approach as most students have access to a PC and to Excel and so making examples available in Excel would make the subject more accessible than if we restricted ourselves to a specialised PC package.

**Having access to Excel is not mandatory.** You will be able to answer examination questions even if their only contact with Excel has been through the pages of this subject guide. However, we do believe that if you have used the Excel spreadsheets associated with this subject guide, it is much more likely that you will:
• enjoy the subject
• better appreciate its applicability to real world decision-making.

Excel spreadsheets for the course

The Excel spreadsheets associated with this subject guide can be accessed and downloaded from the VLE. See the section earlier on in the Introduction for more information on this.

These Excel spreadsheets are updated on an intermittent basis. If you have difficulty in accessing/downloading these spreadsheets please email uolip.vle@lse.ac.uk. Please do not contact the author of this subject guide direct.

Throughout this subject guide we have had to assume that the reader has some familiarity with Excel. One feature of Excel that readers may not have met however is the use of Solver – which is an Excel Tool for solving linear and non-linear programs. Solver is what is referred to as an Excel ‘add-in’ – it adds to the functionality of Excel.

The installation and use of Solver in Excel depends upon the particular version of Excel that you use. Rather than attempt to give a myriad of different instructions here, each dependent on a particular Excel version, we shall simply say that Solver is a component that comes with Excel (itself part of Microsoft Office), but not everybody chooses to install Solver when they install Excel/Office. If you do not have Solver installed then type ‘Solver’ into the Excel help function and you will find some guidance on how to install it.

You may wish to check now whether the PC you use has Solver installed as part of Excel or not. Solver is used in this subject guide in Chapters 6, 8, 9 and 10.

After you have downloaded the Excel spreadsheets associated with this subject guide from the VLE you will see that certain items within them are coloured in red using a bold italic font. These are the items that you are free to change for any examples of your own. All other items (coloured in black) cannot be changed.

Some of you may go to the lengths of examining the underlying Excel coding (i.e. the formulae and cell references used) in these spreadsheets. There is no need to do this. You are not expected to be able to reproduce these spreadsheets yourself, merely to be able to use those provided to you as part of this subject guide intelligently and to interpret the numeric solution values they give you.

Concluding remarks

Even a brief glance at the textbooks associated with this subject guide reveals that there is much more to OR than we have explicitly considered here. Inevitably in producing a subject guide of this type some topics have to be excluded, either because they are less important, or because they are better taught and appreciated using a more interactive approach involving PC software and face-to-face tuition.

Nevertheless we believe that there is sufficient material in this subject guide for you to have gained a clear idea of what OR is about and its value in improving decision making.

We wish you well in your examination and in applying OR to improve the quality of decision making in your future career!
### List of abbreviations used in this subject guide

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AHP</td>
<td>analytic hierarchy process</td>
</tr>
<tr>
<td>AOA</td>
<td>activity on arc</td>
</tr>
<tr>
<td>AON</td>
<td>activity on node</td>
</tr>
<tr>
<td>BOM</td>
<td>bill of materials</td>
</tr>
<tr>
<td>CATWOE</td>
<td>customer, actors, transformation process, worldview, owner, environmental constraints</td>
</tr>
<tr>
<td>CI</td>
<td>consistency index</td>
</tr>
<tr>
<td>CPM</td>
<td>critical path management</td>
</tr>
<tr>
<td>DEA</td>
<td>data envelopment analysis</td>
</tr>
<tr>
<td>DMU</td>
<td>decision making unit</td>
</tr>
<tr>
<td>EBQ</td>
<td>economic batch quantity</td>
</tr>
<tr>
<td>EMV</td>
<td>expected monetary value</td>
</tr>
<tr>
<td>EOQ</td>
<td>economic order quantity</td>
</tr>
<tr>
<td>FCFS</td>
<td>first-come first-served</td>
</tr>
<tr>
<td>FIFO</td>
<td>first-in, first-out</td>
</tr>
<tr>
<td>GP</td>
<td>goal program or goal programming</td>
</tr>
<tr>
<td>IP</td>
<td>integer program or integer programming</td>
</tr>
<tr>
<td>JIT</td>
<td>just-in-time</td>
</tr>
<tr>
<td>JOURNEY</td>
<td>JOintly Understanding, Reflecting, and NEgotiating strategY</td>
</tr>
<tr>
<td>LIFO</td>
<td>last-in, first-out</td>
</tr>
<tr>
<td>LP</td>
<td>linear program or linear programming</td>
</tr>
<tr>
<td>MAUA</td>
<td>multi-attribute utility analysis</td>
</tr>
<tr>
<td>MAVA</td>
<td>multi-attribute value analysis</td>
</tr>
<tr>
<td>MCDA</td>
<td>multicriteria decision analysis</td>
</tr>
<tr>
<td>MIP</td>
<td>mixed-integer program or mixed-integer programming</td>
</tr>
<tr>
<td>MRP</td>
<td>materials requirements planning</td>
</tr>
<tr>
<td>MS</td>
<td>management science</td>
</tr>
<tr>
<td>OPT</td>
<td>optimised production technology</td>
</tr>
<tr>
<td>OR</td>
<td>operational/operations research</td>
</tr>
<tr>
<td>PERT</td>
<td>program evaluation and review technique</td>
</tr>
<tr>
<td>PSM</td>
<td>problem structuring methods</td>
</tr>
<tr>
<td>RD</td>
<td>root definition</td>
</tr>
<tr>
<td>SC</td>
<td>strategic choice</td>
</tr>
<tr>
<td>SCM</td>
<td>supply chain management</td>
</tr>
<tr>
<td>SODA</td>
<td>strategic options development and analysis</td>
</tr>
<tr>
<td>SSM</td>
<td>soft systems methodology</td>
</tr>
<tr>
<td>UE</td>
<td>uncertainty about the working environment</td>
</tr>
<tr>
<td>UR</td>
<td>uncertainty about related decision fields</td>
</tr>
<tr>
<td>UV</td>
<td>uncertainty about guiding values</td>
</tr>
</tbody>
</table>
Notes
Chapter 1: Methodology

Essential reading
Anderson, Chapter 1, section start–1.4.

Aims of the chapter
The aims of this chapter are to:
• give a brief introduction to the evolution of OR
• illustrate OR by considering a very simple decision problem
• illustrate translating an imprecise verbal description of a problem into a precise mathematical description
• discuss some methodological issues that arise in OR work.

Learning outcomes
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:
• explain the philosophy underlying the reasons for mathematical modelling of problems
• describe the phases of an OR project
• explain the philosophy underlying OR
• explain how OR is carried out (i.e. the client/consultant role)
• discuss consultancy, cost versus decision quality, optimisation and implementation in the context of OR work
• discuss the benefits of an OR approach to decision problems.

Introduction
We hope in this chapter to illustrate to you that decision-making situations can be transformed from a (perhaps imprecise) verbal description to a precise mathematical description. This transformation, although involving the use of mathematics, does not usually demand a high level of mathematical skill. The chapter begins with a brief introduction to the evolution of OR. We then actually do some OR by considering a very simple decision problem. We highlight some general lessons and concepts from this specific example. We then discuss some methodological issues that arise in OR work.

I would like to emphasise here that OR is (in my view) a subject/discipline that has much to offer in making a difference in the real world. OR can help you to make better decisions and it is clear that there are many, many people and companies out there who need to make better decisions.

Evolution
OR is a relatively new discipline. Whereas in 1930 it would have been possible to study mathematics, physics or engineering (for example) at university, it would not have been possible to study OR; indeed, the term OR did not exist then. It started in the UK as an organised form of research
just before the outbreak of the Second World War in 1939. Scientists were attempting to make operational use of radar data (radar only just having been developed) for the air defence of the UK. The term ‘operational research’ (RESEARCH into (military) OPERATIONS) was coined as a suitable description for this new branch of applied science.

During the Second World War, OR developed both in the UK and in the USA and was used in many different situations to help determine effective operational methods (e.g. how large convoys carrying food and other supplies across the Atlantic should be organised to minimise the number of ships lost). By the end of the war in 1945 OR was well established in the armed services in both the UK and the USA.

Although scientists had (plainly) been involved in the hardware side of warfare (designing better planes, bombs, tanks, etc), scientific analysis of the operational use of military resources had never taken place in a systematic fashion before the Second World War. Military personnel were simply not trained to undertake such analysis.

These early OR workers came from many different disciplines; one UK group consisted of a physicist, two physiologists, two mathematical physicists and a surveyor. What such people brought to their work were ‘scientifically trained’ minds, used to querying assumptions, logic, exploring hypotheses, devising experiments, collecting data, analysing numbers, etc. Many too were of high intellectual calibre (at least four UK wartime OR personnel were later to win Nobel prizes when they returned to their peacetime disciplines).

Features of this early OR work were:

• the scientific basis of the work and of the people involved in doing it
• work was carried out by a team of individuals, that team often being made up of individuals from different scientific disciplines
• work was organised into projects (specific pieces of work with explicit terms of reference to be completed in a set time)
• the relationship between the OR worker (or team) and the decision maker, where the OR worker/team carried out the project but the decision maker implemented any solution and bore responsibility for its success or failure
• the use of data collection to develop an understanding of the problem under investigation
• the need for OR workers to work with all ranks (both junior and senior) within the organisation.

Many of these features are still present in current OR work. One feature that has (inevitably) decayed over time, however, is that as the subject knowledge base of OR has expanded, present-day OR teams typically do not include individuals from different scientific disciplines. Instead a team might well contain individuals who have received some specialised university level education (at undergraduate or Masters level) in OR.

In 1945, following the end of the war, OR took a different course in the UK to that in the USA. In the UK many of the OR workers returned to their original peacetime academic disciplines. As such, OR did not spread particularly well, except for a few isolated industries (iron/steel and coal). In the USA, OR spread to the universities so that systematic training in OR for future workers began. Nowadays of course OR can be found worldwide.
It is perhaps worth stating here that activities that would, in modern light, be viewed as OR had occurred before the 1930s. For example the Economic Order Quantity formula (dealt with in Chapter 5) which helps decide how much stock a company should order from a supplier is believed to date from the early 1900s. However, it was only from the 1930s onwards that OR really established itself as a recognised professional activity and as a coherent scientific discipline.

**Activity**

Explore the internet to see if universities in your own country offer courses in operations research or management science.

A very recent evolution in terms of OR has been the appearance of the term **analytics** (or data analytics) that deals with the same issues as we consider in OR. Other phrases associated with analytics are ‘data science’ and ‘data scientist’. Often analytics is associated with ‘big data’, where the volume of data to be considered is large.

**Activity**

Explore the internet to gain an understanding of the following terms:

- analytics
- big data
- data science
- data scientist.

In order to get a clearer idea of what OR is we shall actually do some by considering the specific problem below and then highlight some general lessons and concepts from this specific example. Note that you should be clear here that we are not suggesting that all of OR can be equated to what you are about to read below. Rather we are using a simple example to introduce you to the type of approach that may be followed when tackling a decision problem using OR.

**Be clear here – what you are about to read below is a very different approach to decision making to anything you will have encountered before. You may well find it intellectually demanding and (initially at least) hard to follow. We make no apologies for this.**

**Two Mines company**

The Two Mines company own two different mines that produce an ore which, after being crushed, is graded into three classes: high, medium and low-grade. The company has contracted to provide a smelting plant with 12 tons of high-grade, 8 tons of medium-grade and 24 tons of low-grade ore per week. The two mines have different operating characteristics as detailed below.

<table>
<thead>
<tr>
<th>Mine</th>
<th>Cost per day (£'000)</th>
<th>Production (tons/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>High</td>
</tr>
<tr>
<td>X</td>
<td>180</td>
<td>6</td>
</tr>
<tr>
<td>Y</td>
<td>160</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 1.1:** Two Mines company.

How many days per week should each mine be operated to fulfil the smelting plant contract?
Note:

- This is clearly a very simple (even simplistic) example but, as with many things, we have to start at a simple level in order to progress to a more complicated level.
- This is a decision problem (we have to decide something); many of the techniques/topics you will meet in this subject guide address decision problems.

Activity

Consider this problem by yourself for 10 minutes. What answer do you come up with for the number of days per week each mine should be operated? What is the associated cost? Write your answer here for later reference.

Guessing

To explore the Two Mines problem further we might simply guess (i.e. use our (managerial) judgement) how many days per week to work and see how any guesses we make work.

Work one day a week on X, one day a week on Y

This does not seem like a good guess as it results in only 7 tonnes a day of high-grade, insufficient to meet the contract requirement for 12 tonnes of high-grade a day. We say that such a solution is infeasible.

Work 4 days a week on X, 3 days a week on Y

This seems like a better guess as it results in sufficient ore to meet the contract. We say that such a solution is feasible. However, it is quite expensive (costly).

Rather than continue guessing we can approach the problem in a structured logical fashion as below. Ideally we would like a solution that supplies what is necessary under the contract at minimum cost. Logically such a minimum cost solution to this decision problem must exist. However, even if we keep guessing we can never be sure whether we have found this minimum cost solution or not. Fortunately our structured approach will enable us to find the minimum cost solution.

Two Mines solution

What we have is a verbal description of the Two Mines problem. What we need to do is to translate that verbal description into an equivalent mathematical description. In dealing with problems of this kind we often do best to consider them in the order:

- variables
- constraints
- objective.

We do this below. Please note here that this process is often called formulating the problem (or more strictly formulating a mathematical representation of the problem).

Variables

These represent the ‘decisions that have to be made’ or the ‘unknowns’.

Let:

\[ x = \text{number of days per week mine X is operated} \]

\[ y = \text{number of days per week mine Y is operated} \]

Note here that \( x \geq 0 \) and \( y \geq 0 \).


**Constraints**

It is best to first put each constraint into words and then express it in a mathematical form.

- **Ore production constraints** – balance the amount produced with the quantity required under the smelting plant contract.
  - High \(6x + 1y \geq 12\)
  - Medium \(3x + 1y \geq 8\)
  - Low \(4x + 6y \geq 24\)

- Note we have an inequality here rather than an equality. This implies that we may produce more of some grade of ore than we need. In fact, we have the general rule: **given a choice between an equality and an inequality, choose the inequality**.

  - For example – if we choose an equality for the ore production constraints we have the three equations \(6x + y = 12\), \(3x + y = 8\) and \(4x + 6y = 24\) and there are **no** values of \(x\) and \(y\) which satisfy all three equations (the problem is therefore said to be ‘over-constrained’). For example, the values of \(x\) and \(y\) which satisfy \(6x + y = 12\) and \(3x + y = 8\) are \(x = 4/3\) and \(y = 4\), but these values do not satisfy \(4x + 6y = 24\).

- The reason for this general rule is that choosing an inequality rather than an equality gives us more flexibility in optimising (maximising or minimising) the objective (deciding values for the decision variables that optimise the objective).

- **Days per week constraint** – we cannot work more than a certain maximum number of days a week; for example, for a 5-day week we have:
  - \(x \leq 5\)
  - \(y \leq 5\)

- Constraints of this type are often called **implicit** constraints because they are implicit in the definition of the variables.

**Objective**

Again in words our objective is (presumably) to minimise cost which is given by:

\[180x + 160y\]

Hence we have the complete mathematical representation of the problem as:

\[
\begin{align*}
\text{minimise} & \quad 180x + 160y \\
\text{subject to} & \quad 6x + 1y \geq 12 \\
& \quad 3x + 1y \geq 8 \\
& \quad 4x + 6y \geq 24 \\
& \quad x \leq 5 \\
& \quad y \leq 5 \\
& \quad x, y \geq 0
\end{align*}
\]

**Activity**

Suppose now that there is a third mine \(Z\), costing \(120\) (£’000) per day and producing 0.5 tonnes of high-grade, one tonne of medium-grade and nine tonnes of low-grade ore per day. What would the formulation of the problem now be?
Discussion

There are a number of points to note here:

- A key issue behind formulation is that it makes you think. Even if you never do anything with the mathematics this process of trying to think clearly and logically about a problem can be very valuable.

- A common problem with formulation is to overlook some constraints or variables and the entire formulation process should be regarded as an iterative one (iterating back and forth between variables/constraints/objective until we are satisfied).

- The mathematical problem given above has the form:
  - all variables continuous (i.e. can take fractional values)
  - a single objective (maximise or minimise)
  - the objective and constraints are linear i.e. any term is either a constant or a constant multiplied by an unknown (e.g. 24, 4x, 6y are linear terms but xy is a non-linear term).

- Any formulation which satisfies these three conditions is called a linear program (LP). As we shall see later LPs are important.

- We have (implicitly) assumed that it is permissible to work in fractions of days – problems where this is not permissible and variables must take integer values and will be dealt with under integer programming (IP).

- Often (strictly) the decision variables should be integer but for reasons of simplicity we let them be fractional. This is especially relevant in problems where the values of the decision variables are large because any fractional part can then usually be ignored (note that often the data (numbers) that we use in formulating the LP will be inaccurate anyway).

- The way the complete mathematical representation of the problem is set out above is the standard way (with the objective first, then the constraints and finally the reminder that all variables are \( \geq 0 \)).

Considering the Two Mines example given above:

- This was a decision problem.

- We have taken a real-world situation and constructed an equivalent mathematical representation – such a representation is often called a mathematical model of the real-world situation (and the process by which the model is obtained is called formulating the model).

- Just to confuse things the mathematical model of the problem is sometimes called the formulation of the problem.

- Having obtained our mathematical model we (hopefully) have some quantitative method which will enable us to numerically solve the model (i.e. obtain a numeric solution) – such a quantitative method is often called an algorithm for solving the model. Essentially an algorithm (for a particular model) is a set of instructions which, when followed in a step-by-step fashion, will produce a numeric solution to that model. Many algorithms for OR problems are available in computer packages.

- Our model has an objective, that is something which we are trying to optimise.

- Having obtained the numeric solution of our model we have to translate that solution back into the real-world situation.
Hence we have a definition of OR as:

**OR is the representation of real-world systems by mathematical models together with the use of quantitative methods (algorithms) for solving such models, with a view to optimising them.**

**Activity**

Think of a number of real-world business systems of which you are aware. Do you see scope for OR to make a difference to those systems or not?

**Philosophy**

In general terms we can regard OR as being the application of scientific methods/thinking to decision making. Underlying OR is the philosophy that:

- decisions have to be made
- using a quantitative (explicit, articulated) approach will lead (on average) to better decisions than using non-quantitative (implicit, unarticulated) approaches (such as those used by human decision makers).

Indeed it can be argued that although OR is imperfect it offers the best available approach to making a particular decision in many instances (which is not to say that using OR will produce the right decision).

Often the human approach to decision making can be characterised (conceptually) as the ‘ask Fred’ approach: simply give Fred (‘the expert’) the problem and relevant data, shut him in a room for a while and wait for an answer to appear.

The difficulties with this approach are:

- speed (cost) involved in arriving at a solution
- quality of solution – does Fred produce a good quality solution in any particular case
- consistency of solution – does Fred always produce solutions of the same quality (this is especially important when comparing different options).

You can form your own judgement as to whether OR is better than this approach or not.

**Activity**

Form your own judgement as to whether OR is better than the ‘ask Fred’ approach or not. Can you think of problems you have solved using the ‘ask Fred’ approach?

**Activity**

What do you think is the minimum cost solution to the Two Mines problem? Record it here for reference.

**Certainty versus uncertainty**

In management decision making it is helpful to consider whether we are deciding under conditions of certainty or uncertainty. The Two Mines problem considered above was one where we had certainty, we knew all data values precisely. Uncertainty can arise for two basic reasons:

- we are not sure of the exact numeric value for a data item
• probability (chance) plays a natural role in the decision problem.

As to the first of these, this might occur if we were unsure as to the precise cost per day of operating mines X and Y in the Two Mines problem and only had imprecise information as to these costs. As to the second of these, this might occur if we had a decision problem relating to the price to bid on a contract and we are unsure, for a given price, of the probability that our bid would be accepted.

It is sometimes believed in management that decision problems only arise due to uncertainty. **This is not true.** As the Two Mines problem indicates it may be difficult to make a decision even when we are absolutely certain about everything. For example, do you know what the minimum cost solution to that problem is? If you think you do, would you be prepared to bet your life and that of your entire extended family on being correct?

Uncertainty, it is true, does tend to complicate a problem, but you will see many examples throughout this subject guide where we are certain about the problem being considered and all data items, but still have a difficult decision problem to solve.

### Phases of an OR project

Drawing on our experience with the Two Mines problem we can identify the phases that a (real-world) OR project **might** go through. We are not suggesting here that all OR projects go through the phases shown below, rather that the phases shown are a sufficiently good description of the phases that many projects go through to merit consideration.

#### Phase 1: Problem identification

In this phase we attempt to clarify the problem that we have to consider. Factors that need consideration here are:

• Diagnosis of the problem from its symptoms if not obvious (i.e. what is the problem?).

• Delineation of the sub-problem to be studied. Often we have to ignore parts of the entire problem as it is simply too large/complex/time-consuming to be tackled. Rather we choose some distinct part of the overall problem and tackle that. Be aware here that often benefit can be gained just by making improved decisions for some part of an overall problem.

• Establishment of objectives, limitations and requirements for the problem (or part of the problem) we have chosen to tackle.

#### Phase 2: Formulation as a mathematical model

It may be that a problem can be modelled in differing ways, and the choice as to the appropriate model may be crucial to the success of the OR project. In addition to algorithmic considerations for solving the model (i.e. can we solve our model numerically?) we must also consider the availability and accuracy of the real-world data that are required as input to the model.

Note that the **data barrier** (‘we do not have the data!!!’) can appear here, particularly if people are trying to block the project. Often data can be collected or estimated, particularly if the potential benefits from the project are large enough.

You will also find, if you do much OR in the real world, that some environments are naturally **data-poor**, that is the data are of poor quality or non-existent, and some environments are naturally **data-rich**.
This issue of the data environment can affect the model that you build. If you believe that certain data can never (realistically) be obtained there is perhaps little point in building a model that uses such data.

**Activity**

Have you ever met the data barrier in your own work? Think of an environment which you consider to be data-poor and an environment which you consider to be data-rich.

**Phase 3: Model validation (or algorithm validation)**

Model validation involves running the algorithm for the model on the computer in order to ensure:

- the input data are free from errors
- the computer program is bug-free (or at least there are no outstanding bugs)
- the computer program correctly represents the model we are attempting to validate
- the results from the algorithm seem reasonable (or if they are surprising we can at least understand why they are surprising).

Sometimes we feed the algorithm historical input data (if available and relevant) and compare the output with the historical result.

**Phase 4: Solution of the model**

Standard computer packages, or specially developed algorithms, can be used to solve the model (as mentioned above). In practice, a ‘solution’ often involves considering different numeric scenarios under varying assumptions to establish sensitivity. For example, what if we vary the input data (which will be inaccurate anyway), then how will this affect the values of the decision variables? Questions of this type are commonly known as 'what if' questions nowadays.

Note here that the factors which allow such questions to be asked and answered are:

- the speed of processing (turn-around time) available by using PCs
- the interactive/user-friendly nature of many PC software packages.

**Activity**

Think of any decision you may have made based on analysing numbers. Did you conduct sensitivity analysis to see if your decision would be different if the numbers changed or not? If not, why not?

**Phase 5: Implementation**

This phase is **implementation**: that is, making a difference (hopefully for the better!) in the real world.

It may involve the implementation of the results of the study or the implementation of the **algorithm** for solving the model as an operational tool (usually in a computer package). In the first instance detailed instructions as to what has to be done (including time schedules) to implement the results must be issued. In the second instance operating manuals and training schemes will have to be produced for the effective use of the algorithm as an operational tool.
Note here that although we have presented the five phases above in a sequential fashion, in practice we might well switch between phases as and when the need dictates. For example we might find at Phase 4 when we examine numeric solution values that we have made an error in our formulation of a mathematical model at Phase 2 and so we need to loop back to that phase.

It is believed that many OR projects that successfully pass through the first four phases given above fail at Phase 5, the implementation stage (i.e. the work that has been done does not have a lasting effect). As a result one topic that has received attention in terms of bringing an OR project to a successful conclusion (in terms of implementation) is the issue of client involvement. This means keeping the client (the sponsor/originator of the project) informed and consulted during the course of the project so that they come to identify with the project and want it to succeed. Achieving this is really a matter of experience. However, we believe that, as with many things in life, some useful insights can be gained from the written word (as opposed to real-life experience) and for this reason we discuss this issue of implementation further below.

Methodological issues

There are a number of methodological issues that arise in OR work that we need to consider here. These relate to:

- consultancy
- cost versus decision quality
- optimisation
- implementation.

We discuss each in turn below.

Consultancy

It often happens that there is a client who has some problem on which they need help and they decide to call in an ‘expert’ to provide that help. The ‘expert’ is called a consultant and the process in which they engage (tackling the client’s problem) is called consultancy. Clearly a client might well engage an OR worker as a consultant (for example, because the OR worker has skills that the client lacks). Clients can be drawn from a wide range of organisations (for example, private companies, public companies, and governmental departments).

There is no consultant without a client, and the consultant needs to be clear who the client is, the nature of the problem, and what kind of help is needed. Often the answer to these questions will be covered in some form of contract between the client and consultant. However, this may change over the course of the project and therefore should be kept under review. Close contact between the client and the consultant is a key determinant of the success of the project and will be discussed further below.

Problems have a number of characteristics:

- things are not as they should be, or understanding is incomplete
- the problem owner wants to do something about it

Activity

Think of a business situation where you were instrumental in making a difference in the real world. Was this difference really for the better or not and why?
• the problem owner either:
  ◦ does not know what to do
  ◦ knows what needs to be done but lacks the time/skills to do it themselves.

The client should be the ‘problem owner’ (that is, the person or group who is responsible and who can make changes). However, it may be that a consultant is called in by another person, and in these circumstances the consultant will need to address the ‘problem owner’ through the client. In addition, the consultant should be aware what changes the client can feasibly make. Some problems cross departmental boundaries, and the client may only be able to make changes in their department.

**Activity**

Think of a problem of which you are aware. Who is the problem owner? Who is the client?

In situations where the client is not the person commissioning the work or where the problem crosses departmental boundaries, the consultant must ensure that they can give advice on the options available so that the client can make changes. Otherwise, the consultant’s work is likely to be in vain.

**Activity**

Can you think of a situation from your experience where a consultant’s work has been in vain? Why did the work not succeed?

The consultant therefore helps the client decide what to do. Defining the problem should involve finding and agreeing some activity that will be useful in helping the client decide what to do. Note that problems are subjective, and therefore so are solutions. It is the client’s problem and the desired solution should be the client’s, not the consultant’s. The consultant must respond to the client’s concerns and value systems, or risk the whole enterprise – these can be debated/negotiated. However, ultimately the client decides.

Clearly the consultant needs to acquire an understanding of the context in which the problem is set. There is usually some obvious technical context that needs to be understood (for example, the client’s organisation is providing services of a particular type, using these particular resources, to particular customers, and it is, for example, a non-profit making organisation). There is also the social context (who is involved/affected and how these things are articulated, how the actors interact with one another, etc.); and a cultural one (what rules and beliefs are core to the client/organisation, what is the power pattern, how do things get done, etc.).

An organisation or individual may employ consultants because:

• they lack the skills within the organisation to find a resolution to the problem: the consultant is the expert
• they lack the time/resources to find a resolution to the problem: the consultant as a hired body or temporary employee
• they need an ‘independent’ person to help resolve the problem: either to act as an arbitrator between two or more groups, or to provide external justification for a decision, or to audit recommendations of an internal project
• they need to be seen to be doing something and employing a consultant/firm of consultants will provide a positive image
they need a *scapegoat*, someone else to blame if things go wrong.

Obviously in any particular case there may be a mix of reasons for employing a consultant rather than any single reason. Organisations may have their own internal group of consultants or they may employ an outside consultancy firm. The internal consultant has many advantages (e.g. shared organisational objectives, familiarity with context, probable ease of working relationships) but may not be seen as independent and may not have special skills. External consultants may be employed as perceptibly independent and for their special technical skills or relevant previous experience (e.g. with similar organisations in other countries), but their value system may differ from the client's (e.g. their goal may be to maximise their own profit).

**Activity**

Has your company (or a former company) ever employed consultants? If so, why did they do so?

**Activity**

Would you be the best person to carry out some consultancy work for your work/college? Why or why not?

**Cost versus decision quality**

Consider a simple decision problem, should you ask someone to marry you or not? You could reach a solution to this decision problem very easily (very quickly and cheaply) by tossing a coin – heads I ask, tails I do not. Alternatively, you might consider it worthwhile spending time and money finding out whether you are compatible before deciding whether or not to ask that person to marry you. The point is perhaps clear – if we wish to reach good quality decisions then we have to take our time and incur costs.

OR projects use resources – the consultant's, the client's, other staff's – and there is always a tension between minimising the cost and time taken to reach a decision and making the 'perfect' decision. This tension comes in the development of the model, in the collection of the data used in the model, and in the accuracy of the results.

Models are representations of the system under investigation, but are simplified representations. The more time, and cost, put into building a model, the more accurately it should represent the system. The model is also likely to be more complex, including more variables or more relationships between variables, which is likely to make it less accessible to the client and likely to take longer to solve on the computer. Therefore there is a trade-off between an accurate model and the cost and time involved in building the model. The same is true for the collection of data used in the model. It is always possible to collect more data, and spend more time validating the data, and the more time and effort spent in this area the more accurate the data are likely to be.

Note here that although the issue of the time taken to solve a model on a computer is less important today than it was in the past (with modern PCs), there are still many applications where solutions to decision problems have to be produced very quickly. An example of this is the airline industry where decisions as to whether or not to sell a potential passenger a ticket, or how to reconfigure the schedule to cope with disruptions to the pre-planned schedule, must be made very quickly.
Optimisation

The purpose of carrying out a project is usually to provide advice to the client on what to do, based on the construction and experimentation with a model. Traditional OR models have used optimisation to determine what is the best action the client should take (i.e. mathematical models where the optimum value of the controllable variables can be determined). Such a model was seen above for the Two Mines problem.

In general, optimisation assumes that:

- The model accurately represents the system, and therefore the optimal solution for the model is also the optimal solution for the system. This may not be the case since models rarely represent accurately the system under investigation.
- There is one objective or, where there is more than one objective, they can be translated into a common unit, usually monetary values: for instance, giving time a monetary value and so being able to optimise over cost and time.
- There is consensus over what the objective of the system is.
- The problem will not change over time (at least in the short-term) and therefore one optimal solution can be found (i.e. the solution given is the best advice available now for the near future).
- All data can be quantified (i.e. assigned numerical values). In some cases it is not possible to quantify some factors, and therefore only qualitative information can be provided.

Activity

Consider any decision problem that you have been involved in. Do you think that enough time and effort was spent in order to reach a good quality decision or not and why?

Activity

Suppose a number of possible road schemes are being considered and the decision on which scheme to choose is affected by many factors such as: the cost of building, the maintenance cost, the number of cars likely to use any new roads, the estimated number of road deaths, and the environmental impact (houses demolished, pollution) of the scheme. Which of these factors can be numerically assessed, and which can be put in monetary terms?

As an alternative to giving the client the optimal solution the consultant could attempt to scan available options (i.e. attempt to look at all the feasible options in the solution space and evaluate their performance based on the various different objectives of the client). Some of the options could be removed from consideration because other options are better overall objectives (i.e. some options are dominated by other options), but those remaining could be presented to the client for them to determine the trade-off between the objectives, and non-quantifiable factors can then be taken into account.

The effects of inaccurate data and minor changes in the systems' environment can be overcome by carrying out sensitivity analysis using the model. This involves changing assumptions and input data in the model to determine how sensitive the solution is to minor, and major, changes in data.

The accuracy and usefulness of a solution not only depend on the model's ability to represent the system and the accuracy of the data, it also
depends on the **robustness** of the solution obtained. A solution is said to be robust if the option identified as being 'best' remains the best option (or if not best at least an extremely good option) even when the situation changes (e.g. environmental changes, such as cost and demand data). More informative and robust results are likely to be obtained by putting extra time into experimentation with the model (e.g. changing input data, changing assumptions about the future environment). It is always important to carry out some sensitivity analysis on the results from a model to check the robustness of the solution, and also to identify the cost of choosing an option which, while it may not be the 'best', is more robust.

**Activity**

Suppose there were two types of electricity generators – one cost £100,000 and was sufficient to supply 20,000 homes, the other cost £150,000 and was sufficient to supply 35,000 homes – if there are currently 40,000 homes to be supplied, how many of each type of generator should be built?

If the generators are each expected to last 10 years, and the number of homes is estimated to rise in five years to 55,000 and then remain stable, how many of each type should be built?

What would be a robust strategy if the number of homes in the next 10 years is likely to be between 50,000 and 60,000?

Many of the models presented in this subject guide use optimisation. You should be aware of the limitations of optimisation and also conscious of the need to carry out sensitivity analysis. We will look at robustness and sensitivity analysis with regard to one of the optimisation techniques, linear programming, in Chapter 8 of this subject guide.

**Activity**

Consider the various problems associated with optimisation and identify the possible strategies for dealing with them.

**Implementation**

To a large extent the success of an OR project is not determined by whether the project produces an elegant model, or by the size of the benefits of the recommended course of action, but by whether the project affects the decisions made by the client, including whether any action recommended is undertaken by the client.

In order for an organisation to implement a proposed course of action, it must be possible to implement the solution (technologically and culturally), and the person(s) with the power to implement the recommended course of action must be committed to it.

In order for an OR project to produce a solution which it is possible to implement (a feasible solution), it is necessary to ensure, when formulating the problem, that all the relevant technological and cultural constraints are known and, where appropriate, included in the model. Continuous contact with the organisation, and specifically the client in the organisation (the person/people who have the problem), including regular discussions on the progress of the project and the model being produced, should ensure that a feasible solution is produced.

Gaining the commitment of the person(s) with the power to implement the solution requires the consultant to persuade them that the changes recommended are worth making. Approaches which are likely to gain such commitment include:
• **Regular and positive communication** with the client, to get information as well as to ensure the problem has not changed, and to make sure that the client knows how the problem is being approached. Problems can change over time; they may go away of their own accord; the objectives of the organisation may change; the constraints may change. A technically brilliant solution to a problem the client no longer faces, or to the wrong problem, will have no value to the client and will not be implemented, or may be impossible to implement.

• Continually **involving the client** during the project. A good practical rule is that the consultant should never surprise the client by what they do – the **result** may surprise the client (i.e. it may not fit with their previously conceived ideas of what the solution would be), but the **work** that is being done should not.

• Giving the solution, and justification for the solution, in plain language, **using terms the client will understand**. This may require the use, at least in persuading the client of the validity of the solution, of simplified, transparent models and possibly examples showing the benefits of the solution. A common criticism of OR work over the years has been that some of the models used are ‘opaque’ (they cannot be understood by the client) which can lead to non-implementation because the client does not believe in the model and therefore cannot confidently implement results, especially where the results are counter-intuitive or very radical.

• Ensuring that the action recommended in the solution is within the **power of the client** (not recommending action which the client is unable to implement).

**Activity**

Reflect on how you might persuade your boss or a former boss to support a course of action you are proposing. What strategies might you adopt?

**Benefits**

Throughout your career you will inevitably encounter people who have little understanding of OR and, moreover, feel that problems in business/management can be solved by (innate) personal ability and experience (such as coincidentally they themselves possess). As such they see no need for a ‘complicated’ approach such as OR.

On a personal note at the time of writing I am over 60 and have been involved in OR for all of my working life. My personal view as to the benefits of OR in solving problems in business/management is:

• OR is particularly well-suited for routine tactical decision-making where data are typically well-defined and decisions for the same problem must be made repeatedly over time (for example, how much stock to order from a supplier).

• An advantage of explicit decision-making is that it is possible to examine assumptions explicitly.

• We might reasonably expect an ‘analytical’ (structured, logical) approach to decision-making to be better (on average) than simply relying on a person’s innate decision-making ability.

• OR techniques combine the ability and experience of many people.

• Sensitivity analysis can be performed in a systematic fashion.
• OR enables problems too large for a person to tackle effectively to be dealt with.
• Constructing an OR model highlights what is/is not important in a problem.
• If you have an explicit OR model then it has the advantage that it is transparent – conceptually you can write it down on a piece of paper and everyone (specifically everyone who has a suitable amount of training) can examine it, discuss it, criticise it and amend it as appropriate.
• A training in OR teaches a person to think about problems in a logical fashion.
• Using standard OR techniques prevents a person having to ‘reinvent the wheel’ each time.
• OR techniques enable computers to be used with (usually) standard packages and consequently all the benefits of computerised analysis (speed, rapid (elapsed) solution time, graphical output, etc.).
• OR techniques are an aid (complement) to ability and experience, not a substitute for them.
• Many OR techniques are simple to understand and apply.
• There have been many successful OR projects.
• Many successful companies use OR techniques.
• Ability and experience are vital. However, OR is necessary to use these effectively in tackling large problems.
• OR techniques free executive time for more creative tasks.

Links to other chapters

The topics considered in this chapter link to all the other chapters in this subject guide. This chapter ‘sets the scene’ for later chapters in that it discusses the discipline of OR and a number of important general issues that arise within it.

Case studies

The case studies associated with this chapter are given below. We would encourage you to read them.

<table>
<thead>
<tr>
<th>Title</th>
<th>Anderson (page number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue management at American Airlines</td>
<td>3</td>
</tr>
<tr>
<td>Workforce scheduling for British Telecommunications PLC</td>
<td>8</td>
</tr>
<tr>
<td>Quantitative analysis at Merrill Lynch</td>
<td>13</td>
</tr>
<tr>
<td>Models in Federal Express</td>
<td>15</td>
</tr>
<tr>
<td>A spreadsheet tool for Catholic Relief Services</td>
<td>18</td>
</tr>
</tbody>
</table>
A reminder of your learning outcomes

Having completed this chapter, and the Essential reading and activities, you should be able to:

• explain the philosophy underlying the reasons for mathematical modelling of problems
• describe the phases of an OR project
• explain the philosophy underlying OR
• explain how OR is carried out (i.e. the client/consultant role)
• discuss consultancy, cost versus decision quality, optimisation and implementation in the context of OR work
• discuss the benefits of an OR approach to decision problems.

Sample examination questions

For sample examination questions relating to the material presented in this chapter please visit the VLE.
Chapter 2: Problem structuring and problem structuring methods

Essential reading
Rosenhead, Chapters 2, 4 and 6.

Aims of the chapter
The aims of this chapter are:
• to contrast hard OR and soft OR (problem structuring methods)
• to illustrate three soft OR approaches:
  ◦ Journey Making
  ◦ Soft Systems Methodology
  ◦ Strategic Choice.

Learning outcomes
By the end of this chapter, and having completed the Essential reading and activities, you should ensure that you can describe and explain:

General
• the common features of problem structuring methods.

Journey Making
• how cognitive maps can structure individual and group views of a problem, including the identification of goals and options, and how to produce and structure a cognitive map
• the Journey Making process.

Soft Systems Methodology (SSM)
• how root definitions and CATWOE are produced and used to describe pure systems
• the seven stage SSM process.

Strategic Choice (SC)
• how uncertainties can be classified and reduced
• the four modes of working and the SC process.

Introduction
The first stage in the OR process, formulation of the problem, involves identifying what is the problem facing the organisation. Rosenhead defines a well-structured problem as one with:
• unambiguous objectives
• firm constraints
• established cause–effect relationships.

The problem formulation stage attempts to identify and clarify these factors.
Formulation of a problem may be difficult because:

- The problem is inter-related, either being one of a number of problems which are facing different parts of an organisation, or the problem itself is made up of a number of inter-related problems.
- There is disagreement within the organisation over the objectives, the constraints and/or cause–effect relationships.
- There is a large amount of uncertainty over the constraints and/or the cause–effect relationships.

Problems that are difficult to formulate are often strategic in nature, and it is for this type of problem that the techniques presented in this chapter are most relevant.

The formulation of such problems is often the most important and difficult part of the OR process, and may in itself yield answers for the problem owner.

Activity
Think of a problem of which you are aware. Write down a clear statement of what the problem is. Did you find getting this clear statement difficult or not and why?

Activity
Consider the different perceptions to the definition of a good lesson by a teacher, students and a school management body.

In recent years a number of approaches to problems have appeared which have come to be labelled collectively as 'soft OR'. Many of these approaches have their origins in the UK.

By contrast the classical OR techniques such as linear programming are labelled collectively as 'hard OR'.

Hard OR is used here in the sense that traditional/classical OR techniques are:

- tangible
- easy to explain
- easy to use.

Soft OR, by contrast, is:

- somewhat intangible
- not easy to explain
- not easy to use.

Note here that a collective generic name for these particular soft OR approaches is problem structuring methods.

This chapter introduces some of the concepts behind problem structuring, and a number of problem structuring methods.

Problem structuring methods

Before going on to consider each of the problem structuring methods (PSMs for short) in detail it will be helpful if we first outline what they have in common.

Probably the main characteristic of such methods is that, to a greater or lesser extent, their primary focus is on:

the people involved with the problem
and their secondary focus is on:

**the problem.**

This is in sharp contrast to traditional hard OR approaches which are geared to understanding the problem and developing the best answer to it. Or to phrase it another way in hard OR the primary focus is on:

**the problem.**

and the secondary focus (if at all) is on:

**the people involved with the problem.**

You can clearly see here how in problem structuring methods the focus is the reverse to that of hard OR.

Although it is plain that both issues need to be considered the focus adopted can lead to some distinct differences.

For example, if my primary focus is on **the people involved with the problem**, I could regard myself as a **success** if I can get them to better understand the issues that they face and to agree on a course of action. The fact that this course of action may be an absolutely disastrous one from the point of view of **the problem** is somewhat irrelevant.

### Overview

As an overview, problem structuring methods:

- help structure (complex) problems
- are mainly used with a small group of decision makers (people) in an organisation
- do not try to get an objective definition of the problem
- emphasise the importance and validity of each individual's subjective perception of the problem.

To achieve this such methods typically use a consultant (external person) whose role is:

- to see that the group contains individuals with knowledge of the situation and/or individuals who will affect the success of any action proposed
- to act as a facilitator/organiser of the process
- to orchestrate discussion
- to be seen to be open, independent and fair.

The consultant does not need to possess any special knowledge about the problem (i.e. he or she does not need to be an expert in the problem area). However, consultants are often experts in the particular problem structuring method being applied.

Such methods try to capture the group's perception of the problem:

- verbally (in words)
- in pictures/diagrams.

Words are used as they are believed to be the natural currency of problem definition/discussion/solution (compare hard OR which uses mathematics). The use of pictures/diagrams helps to structure the group's perception of the problem and enables discussion/debate to be less personal.

Such methods help the members of the group:

- to gain an understanding of the problem they face
• to gain an understanding of the views of the problem adopted by other members of the group
• to negotiate about the action to take
• to agree on a consensus course of action to which they are committed.

Definitions

Problem structuring methods involve the use of a number of words with specific meanings:

• Client(s) – person(s), the group, who face the decision problem and for whom the consultant is working.
• Consultant – person from outside the group who acts as a facilitator.
• Facilitator – an independent person who aids the group by extracting information from them about the problem and organising it. Facilitators also act as a type of chairperson.
• Consensus – gaining the acceptance of all members of a group to a particular view/decision.
• Workshop – group of people working/discussing an issue or issues in a structured way.
• Pure model – model of a system which pursues a pure purpose from a specific point of view.
• Purposeful activity system – a system, possibly hypothetical, in an organisation which has a specific purpose.

In this chapter we consider three problem structuring methods:

• Strategic Options Development and Analysis (SODA) and JOURNEY (JOintly Understanding, Reflecting, and NEgotiating strategY) Making
• Soft Systems Methodology (SSM)
• Strategic Choice (SC).

To try and illustrate these methods we will apply each of them to the following example problem:

Crime is a real problem in this country. We are spending more and more on locking up increasing numbers of people in prisons, yet crime seems to go on rising. Many of those in prison are there for reasons connected with medical problems (e.g. drug addiction, mental illness), yet when they come out of prison these problems are unresolved and so they go straight back to crime. Perhaps the answer is longer prison sentences.

**Strategic options development and analysis (SODA) and JOURNEY (JOintly Understanding, Reflecting, and NEgotiating strategY) Making**

**Activity/Reading**

For this section read Rosenhead, Chapter 2.

Strategic Options Development and Analysis (SODA) and JOURNEY (JOintly Understanding, Reflecting, and NEgotiating strategY) Making are essentially the same at the level at which we consider them in this chapter. The difference between them relates to the emphasis/meaning implied from the words used to describe the technique:
• SODA – Strategic Options Development and Analysis – implies that the emphasis is on developing and analysing options – possible decisions that can be taken and hence the emphasis is on reaching a final decision

• JOURNEY (JOintly Understanding, Reflecting, and NEgotiating strategY) Making – implies that the emphasis is on understanding/reflecting/negotiating – i.e. the process is as important, or more important, than the final decision reached

Since the description ‘Journey Making’ is intended to encompass and replace ‘SODA’ we shall henceforth use the phrase ‘Journey Making’.

In Journey Making we elicit information from members of the group using individual interviews. Such information is represented on cognitive maps. These show:

• concepts that are relevant
• linkages between concepts.

By concept here we typically mean a short phrase capturing some idea. This idea should be an action-oriented idea that is intended to suggest an option for changing the situation. Often the negative (reverse) of the concept is also introduced.

For example, in talking about our crime example above, a concept used may be ‘more prisoners’ with the negative concept being ‘less prisoners’.

To ease concept representation we use three dots to separate positive and negative concepts so that we can capture both concepts in ‘more prisoners...less prisoners’. This is read as ‘more prisoners rather than less prisoners’. As a shorthand we sometimes omit the negative concept and write ‘more prisoners...’ so that the second concept is implied as the negative of ‘more prisoners’.

Concepts are linked by arrows, with the direction of the arrow being such that concepts representing options lead to concepts representing outcomes. A negative sign associated with an arrow indicates that if the first phrase of the first concept applies, then the second phrase of the second concept also applies.

For example:

```
less crime ...

more prisoners...less prisoners

reform criminals...

longer prison sentences...
```

**Figure 2.1**

Figure 2.1 represents the view that more prisoners links to less crime and longer prison sentences do not reform criminals.

Concepts in maps are generally either:

• goals at the head (the top) of the map (these are things that are self-evidently regarded as ‘good things’)

• options being at the tail (the bottom) of the map.

Strategic options, sometimes called strategic directions, are options which have no other options above them in the map.
In Journey Making, cognitive maps are first produced for each individual by interviewing them in a relatively unstructured ‘free-flowing’ way to try to elicit their thought processes about the problem under discussion and what they think is important about the problem. Such maps often contain 40 to 100 concepts and may also help each individual to refine their thinking.

In Figure 2.2 we show a small map based on our crime example given above. You can see that the goals (at the top of the map) are ‘less crime’ and ‘reform criminals’ and we have a number of options available, e.g. ‘spend more money on prisons’.

Another map for an individual talking about the same problem might be:

For example we might merge our two individual maps above to get:

---

**Activity**

Map your own views about the treatment of crime and prisoners in your society.

Map the views of a friend about the treatment of crime and prisoners in your society.
less crime ...

more medical care in prisons...

longer prison sentences...

reform criminals...

protect society...

more prisoners...less prisoners

spend more money on prisons...

more police...

Figure 2.4

In order to make this map manageable in problems larger than our simple example considered here:

- The concepts in it are aggregated into clusters (say 15 to 30 concepts in each cluster), so that we have a map within each cluster and each cluster is appropriately labelled.
- The final merged map is an overview map at the cluster level showing the labelled clusters and the links between clusters.

Activity

Merge together the two maps you have produced in the preceding two activities.

This merged overview map (and the individual cluster maps) serve as a focus for discussion at a workshop involving:

- analysis of its content and structure
- identification of any ‘emerging themes’ and ‘core concepts’
- discussion of key goals, inter-related problems, key options and assumptions.

As for all problem structuring methods the aim of Journey Making is to achieve understanding/agreement within the group.

Activity

Think of a problem of your own and with a friend apply Journey Making to the problem.

Soft systems methodology (SSM)

Activity/Reading

For this section read Rosenhead, Chapter 4.

SSM assumes:

- Different individuals and groups make different evaluations of events and this leads to them taking different actions.
- Concepts and ideas from systems engineering are useful.
- It is necessary when describing any human activity system to take account of the particular image of the world underlying the description of the system and it is necessary to be explicit about the assumptions underlying this image.
• It is possible to learn about a system by comparing pure models of that system with perceptions of what is happening in the real-world problem situation.

Overview
SSM operates by defining systems of purposeful activity (the root definition), building models of a number of relevant systems, and comparing these models to the real world action going on, in order to structure a debate focusing on the differences. That debate should lead the group of people involved in the process to see their way to possible changes and to motivate these people to carry out these changes.

Stages
There are seven stages in the SSM process, but they are not necessarily followed in a linear fashion. Diagrammatically these stages are:

1. Enter situation considered problematic
2. Express the problem situation
3. Define possible changes which are both desirable and feasible
4. Compare models with real-world situation
5. Build conceptual models of the systems named in the root definitions
6. Formulate root definitions of relevant systems of purposeful activity
7. Take action to improve the problem situation

Figure 2.5

Stages 1 and 2: Finding out
This stage involves entering the problem situation and identifying within it:
• people – essentially all those with an interest in the system or who are likely to be affected by changes to it
• culture – social roles, norms of behaviour, values
• politics – commodities of power and how they are obtained, used, preserved and transmitted.

Stage 3: Developing root definitions
SSM requires one or more root definitions to be stated. These are sentences which describe the ideal system (or subsystems within the overall system).

To ensure that appropriate elements of the system are captured in a root definition it should be possible to deduce from the root definition answers to the following questions:

C Who are the customers/victims/beneficiaries of the system?
A Who are the actors/participants in the system?
T What is transformed by this system; what inputs are transformed into what outputs?
Weltanschauung is a German word for which the usual translation is worldview. It is helpful to consider it as the stock of images in our head, put there by our origins, upbringing and experience of the world. We use these images to help us to make sense of the world and they normally go unquestioned. So what is the worldview underlying the system?

O Who is the owner of the system; who has the power to stop the system?
E What are the environmental constraints that cannot be altered and which need to be considered?

These questions are collectively referred to using the CATWOE mnemonic:

C Customer
A Actors
T Transformation process
W Weltanschauung or worldview
O Owner
E Environmental constraints.

Hence you should be clear here that the root definition and CATWOE are linked together.

So using our crime example a possible root definition could be:

The prison system is a system for ensuring convicted criminals (prisoners) serve their sentences in humane conditions, receive appropriate medical care, are given opportunities to learn training and skills, and are released back into society at the end of their sentence with appropriate support so that they can be reformed from their life of crime.

Checking this root definition against CATWOE to ensure it is appropriate we have:

C Customer – society
A Actors – prisoners and prison staff
T Transformation process – (here it can be regarded as) transforming the need for convicted criminals to be locked away from society to that need being met
W Weltanschauung (worldview) – the desire to reform criminals and to prevent future crime
O Owner – government
E Environmental constraints – criminals exist.

Note that it may be possible to derive alternative answers for one or more of these CATWOE elements from a root definition. For example, the transformation above could be from unreformed criminals to reformed criminals.

The root definition and CATWOE have at their centre the transformation process (the T in CATWOE): what does the system defined by this root definition do? This can be seen diagrammatically as:

Input → Transformation → Output.

Activity

Think of a number of possible transformations for a football match, for a bus service and for a shop. Use the different actors in the system and the different physical objects to help you (for example, how does the shop transform customers, how does it transform the goods on sale, etc?).
Stage 4: Building conceptual models

In SSM a model is a diagram of activities with links connecting them. It:

- is developed from the root definition
- uses verbs or action statements describing the activities that are needed by the root definition
- links these activities according to logical dependencies, an arrow directed from activity x to activity y, namely
  \[ x \rightarrow y \]
- is equivalent to saying activity y is dependent on activity x having been carried out
- should contain between five and nine activities.

The model should contain a monitoring and control subsystem which monitors:

- the **effectiveness** of the system (is this the **right** thing to do?)
- the **efficacy** of the system (does it **work**?)
- the **efficiency** of the system (does it use the minimum resources necessary?).

In building the model, the measures used for these controls need to be determined.

The process of developing root definitions and models can be followed to expand subsections of the overall model (that is, they can be developed for an individual activity/activities in the main model built). Models encompassing the initial root definition can also be built.

With regard to our root definition given above some activities could be:

- appreciate what constitutes humane conditions
- identify appropriate medical care
- identify training needs
- identify skills to be learnt
- identify appropriate support at the end of the sentence
- reform the criminal.

We could link these as shown in the conceptual model (Figure 2.6, below).

With regard to the monitoring and control subsystem (not shown in the model for simplicity) this should address:

- the effectiveness of the system (is this the right thing to do?) – this could be the number of statements on the need to reform criminals made by the government
- the efficacy of the system (does it work?) – this could be the proportion of criminals reformed
- the efficiency of the system (does it use the minimum resources necessary?) – this could be the number of criminals dealt with per pound spent.
Appreciate what constitutes humane conditions
Identify appropriate medical care
Identify training needs
Identify skills to be learnt
Identify appropriate support at end of sentence
Reform criminal

Figure 2.6

**Stage 5: Comparing models with the real world**

This stage involves comparing the models that have been developed with the real world. A systematic way to do this is by ordered questions, namely for each and every activity and link in the model, ask the following questions:

- Does this happen in the real situation?
- How?
- By what criteria is it judged?
- Is it a subject of concern in the current situation?

This stage is designed to provide structure and substance to an organised debate about improving the current situation.

With reference to our model above, for example, we might identify that we are not making sufficient effort to carry out the activity ‘Identify appropriate medical care.’

Of the soft OR approaches considered in this chapter SSM is (in my view) the most powerful/applicable. Thinking clearly and logically about what constitutes an ideal system and then comparing it to the real world can plainly yield insights and ideas that might enable us to make the real world a little more like our ideal world.

**Stage 6: Identifying changes**

This stage involves identifying changes that could be made to the real world system, changes that appear worth trying, to those participating in the SSM process. These changes need to be **systematically desirable** and **culturally feasible**.

With reference to our model above, for example, we might identify putting more resources into medical assessment/care of prisoners as a change to be made.

**Stage 7: Taking action**

This stage involves putting into practice the most appropriate changes identified in the previous stage.

**Activity**

Think of a problem of your own and apply Soft Systems Methodology to the problem.
Strategic choice (SC)

Activity/Reading
For this section read Rosenhead, Chapter 6.

SC identifies four modes of decision making activity:
• shaping – considering the structure of the decision problems
• designing – considering the possible courses of action
• comparing – comparing the possible courses of action
• choosing – choosing courses of action
with the facilitator identifying when to switch between modes, as appropriate.

A key theme underlying SC is the identification of uncertainty areas.

Uncertainty areas
SC identifies three types of uncertainty:
• Uncertainty about the working Environment (UE), reduced by a technical response (e.g. collecting data, surveys, numeric analysis).
• Uncertainty about guiding Values (UV), reduced by a political response (e.g. clarifying objectives, consulting interest groups, asking higher authorities for their opinions).
• Uncertainty about Related decision fields (UR), also known as Uncertainty about choices on Related agendas, reduced by an exploration of structural relationships (e.g. adopting a broader perspective, negotiating/collaborating with other decision makers, looking at the links between a decision that might be made by ourselves and decisions that might be made by others).

Throughout the strategic choice process:
• areas of uncertainty are listed as they arise, and
• are classified by UE/UV/UR.

In the last mode listed above, the choosing mode, these uncertainty areas are addressed in the context of proposed decisions.

Shaping mode
In the shaping mode decision areas are identified as questions. These are simply areas where alternative courses of action are possible (i.e. a choice is possible). These decision areas are then presented on a decision graph, where:
• each area is a node on the graph
• a link (edge) between two nodes (areas) exists if there is thought to be a significant possibility of different outcomes if the two areas are considered separately, rather then together.

Figure 2.7 shows one possible decision graph for our crime example.
**Chapter 2: Problem structuring and problem structuring methods**

**Figure 2.7**

Once the decision graph has been drawn, areas of problem focus – consisting of three or four decision areas – need to be identified. The areas chosen are generally those which are important, urgent and/or connected.

For our crime example above we will have one problem focus based on the areas:

- build more prisons?
- impose longer sentences?
- increase rewards for informing?

With regard to uncertainty we will have just one factor in our uncertainty list, namely:

- Can we find sites to build more prisons? Classified UE.

**Designing mode**

In the designing mode we take each problem focus in turn and:

- List a small number (say two to five) of mutually exclusive possible courses of action (options) in each of the decision areas.
- List incompatible options in different decision areas (note all options in the same decision area are incompatible as they are mutually exclusive); this can be done graphically if so desired using an option graph.
- List (enumerate) all the possible feasible decision schemes where a feasible decision scheme consists of one option from each of the decision areas and none of the options chosen are incompatible.

For our crime example with the problem focus based on the areas:

- build more prisons?
- impose longer sentences?
- increase rewards for informing?

We have the options in each of these decision areas of:

- build more prisons?
  - no
  - yes – five more
  - yes – 10 more
- impose longer sentences?
  - no
  - yes
• increase rewards for informing?
  - no
  - yes

Incompatibilities between options in this example are shown below, where a link in that option graph indicates two options (in different problem areas) that are, in our judgement, incompatible. Note here that this contrasts with the meaning of the lines in the decision graph above – there lines indicated areas that were connected and needed to be considered together – here lines indicate incompatibilities.

**Figure 2.8**

With three options in one decision area and two in the other two there are \(3(2)(2) = 12\) possible decision schemes, although some of these will not be feasible as they involve incompatible options.

These 12 schemes are listed below:

<table>
<thead>
<tr>
<th>Build more prisons?</th>
<th>Impose longer sentences?</th>
<th>Increase rewards for informing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>yes – five more</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>yes – five more</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>yes – 10 more</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>yes – 10 more</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>yes – 10 more</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

**Table 2.1**

Checking each of these schemes we find that for this example there are just three possible feasible decision schemes (labelled A, B and C below) which are:

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Build more prisons?</th>
<th>Impose longer sentences?</th>
<th>Increase rewards for informing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>B</td>
<td>yes – five more</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>C</td>
<td>yes – 10 more</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

**Table 2.2**
With regard to uncertainty, our uncertainty list, after the addition of two more factors, becomes:

- Can we find sites to build more prisons at? Classified UE.
- Will 10 prisons be too many? Classified UE.
- Will the government/judiciary support longer sentences? Classified UV.

**Comparing mode**

In the comparing mode we compare each of the feasible decision schemes. This is done by:

- identifying comparison areas
- within each area, assigning each feasible decision scheme a value.

The values chosen can be monetary sums or values chosen from some scale (e.g. rank on a scale from 1 to 10).

Based on this assignment of values particular schemes may be selected for closer analysis, either individually or as members of a shortlist. A common approach is to compare, in a pairwise fashion, all members of the shortlist. In this pairwise comparison the uncertainty areas are explicitly considered to identify those uncertainty areas relating to the schemes being compared.

For our crime example we could compare our feasible decision schemes (three in this case) with respect to the comparison areas of:

- capital cost (in £’million terms)
- running cost (in £’million terms)
- acceptability to government (from 1 (almost unacceptable) to 5 (neutral) to 10 (very acceptable))
- acceptability to the public (from 1 (almost unacceptable) to 5 (neutral) to 10 (very acceptable)).

We present these numbers below:

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Capital cost</th>
<th>Running cost</th>
<th>Government acceptability</th>
<th>Public acceptability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>200</td>
<td>40</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>400</td>
<td>75</td>
<td>9</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 2.3**

Selecting schemes A (no more prisons, no longer sentences and no increased rewards for informing) and B (five more prisons, longer sentences and increased rewards for informing) for pairwise comparison we have:

<table>
<thead>
<tr>
<th></th>
<th>Scheme A</th>
<th>Scheme B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital cost</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>Running cost</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>Government acceptability</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Public acceptability</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

**Table 2.4**
Recalling the uncertainty list:

- Can we find sites to build more prisons? Classified UE.
- Will 10 prisons be too many? Classified UE.
- Will the government/judiciary support longer sentences? Classified UV.

The only relevant uncertainties for this pairwise comparison of schemes A and B relate to scheme B (five more prisons, longer sentences and increased rewards for informing) and are:

- Can we find sites to build more prisons? Classified UE.
- Will the government/judiciary support longer sentences? Classified UV.

These uncertainty areas are hence ones that will need attention if we are to make a choice between these two schemes. Recall here that all feasible decision schemes, three in this example, are mutually exclusive by the way they were constructed and so, at the end of the process, a choice of just one of them can be made.

**Activity**

Given four schemes, labelled A to D, what pairs of comparisons would be made?

**Choosing mode**

In the choosing mode a commitment package (i.e. what we are proposing to do) is decided upon (or more than one package for submission to higher authorities). A commitment package is guided by the preferred feasible decision scheme and consists of:

- decisions taken now
- explorations to reduce levels of uncertainty (together with estimates of resources needed and timescales)
- decisions deferred until later
- any contingency plans.

With regard to our crime example if we assume scheme B (five more prisons, longer sentences and increased rewards for informing) is the preferred feasible decision scheme then the relevant uncertainty areas are:

- Can we find sites to build more prisons? Classified UE.
- Will the government/judiciary support longer sentences? Classified UV.

The commitment package might be:

- decisions taken now – none
- explorations
  - study to identify provisional sites for five prisons (costing £1,000,000 and taking three months)
  - consult government/judiciary about support for longer sentences (negligible cost, likely to take up to six months)
- decisions deferred – final decision on scheme B until explorations to reduce levels of uncertainty completed
- contingency plans – none.

Note that the actual decision scheme that we choose may be altered by the results of the explorations (for example if our explorations reveal there are no sites available for five prisons).
Choosing and applying PSMs

Suppose that you are faced with a situation that you think can be tackled using a PSM. Which PSM should you choose? Probably the simplest answer is that you should choose the PSM with which you are most familiar. One aspect of PSMs is that ‘success’, however defined, depends significantly on the skill/knowledge/experience of the facilitator who guides the process.

Obviously applying any technique, whether a PSM or one of the more quantitative techniques considered later in this subject guide, is always a matter of skill/knowledge/experience. The question is how much time and effort is required to gain that skill/knowledge/experience. My view would be that the time and effort required is significantly longer for a PSM than for many of the more quantitative techniques considered in later chapters (probably at least an order of magnitude longer).

Suppose though that you are equally experienced (or inexperienced) in the three PSMs considered in this chapter – which one should you choose? My advice would be to choose SSM. In my view the process of thinking clearly about the ‘ideal world’ that is inherent in SSM and then comparing that ideal world against the real world is a powerful one.

Education

You will find that the majority of the topics in this subject guide deal with quantitative/analytic topics. This chapter is the only main chapter that is predominantly qualitative in nature. Hence it is natural to ask whether you, as a student, perhaps quantitatively skilled, should just focus on quantitative topics and miss this chapter out completely in terms of engaging with this subject guide. Clearly that could be done. However, you need to be clear that the purpose of this subject guide is not only to prepare you for the examination, it is also to educate you. Obviously that education is assessed by a single examination in terms of the University of London, but you should be aware that education is for life. Even though you may not focus on this chapter in terms of preparing for the examination that does not mean it is of no value. After you graduate how many years of working life do you think you will face? Anticipating being a millionaire and living a life of leisure by the age of 35? Dream on! Like most of us, myself included, you will work hard all your life – ‘life is hard and then you die’ to quote the phrase. During all those years of working life maybe you will use some of the quantitative topics which you engaged with for the examination. Equally, knowing that problem structuring methods exist, and having some knowledge of what they deal with may, over those working years, be valuable at some point. Time will tell.

Links to other chapters

The topics considered in this chapter do not directly link to other chapters in this subject guide. This chapter introduced methods that can be used when the problem to be faced is essentially qualitative in nature, and a more quantitative (analytic) approach would be inappropriate. Many of the other chapters in this subject guide deal with quantitative topics.
Case studies

The case studies associated with this chapter are given below. We would encourage you to read them.

<table>
<thead>
<tr>
<th>SSM</th>
<th><a href="http://www.learnaboutor.co.uk/strategicProblems/c_s_1frs.htm">www.learnaboutor.co.uk/strategicProblems/c_s_1frs.htm</a></th>
</tr>
</thead>
<tbody>
<tr>
<td>Journey Making</td>
<td><a href="http://www.learnaboutor.co.uk/strategicProblems/c_j_1frs.htm">www.learnaboutor.co.uk/strategicProblems/c_j_1frs.htm</a></td>
</tr>
</tbody>
</table>

A reminder of your learning outcomes

Having completed this chapter, and the Essential reading and activities, you should ensure that you can describe and explain:

General
- the common features of problem structuring methods.

Journey Making
- how cognitive maps can structure individual and group views of a problem, including the identification of goals and options, and how to produce and structure a cognitive map
- the Journey Making process.

Soft Systems Methodology (SSM)
- how root definitions and CATWOE are produced and used to describe pure systems
- the seven stage SSM process.

Strategic Choice (SC)
- how uncertainties can be classified and reduced
- the four modes of working and the SC process.

Sample examination questions

For Sample examination questions relating to the material presented in this chapter please see the VLE.
Chapter 3: Network analysis

Essential reading

Anderson, Chapter 9.

Spreadsheet

network.xls

- Sheet A: Calculation for project completion time
- Sheet B: Calculation for project completion time with delay activity added
- Sheet C: Resource information
- Sheet D: Gantt chart calculated from Sheet C
- Sheet E: Resource usage chart calculated from Sheet C

This spreadsheet can be downloaded from the VLE.

Aims of the chapter

The aims of this chapter are to:

- introduce some simple approaches that can be useful in project management
- give some historical background to these approaches
- illustrate how these approaches can be applied to a simple project
- discuss a number of issues that arise in more complex projects.

Learning outcomes

By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

- draw a network diagram
- calculate the project completion time
- calculate the earliest start time, latest start time and float time for each activity
- identify the critical activities/critical path(s)
- explain the effects of uncertain activity times and cost/time trade-offs
- explain resource smoothing
- explain the benefits of network analysis.

Introduction

Network analysis is the general name given to certain specific techniques which can be used for the planning, management and control of projects. One definition of a project, from the Project Management Institute, is: a temporary endeavour undertaken to create a ‘unique’ product or service.
This definition highlights some essential features of a project:

- it is temporary – it has a beginning and an end
- it is ‘unique’ in some way.

With regard to the use of the word unique I personally prefer to use the idea of ‘non-repetitive’ or ‘non-routine’ (for example, building the very first Boeing Jumbo jet was a project, but building them now is a repetitive/routine manufacturing process).

We can think of many projects in real life (e.g. building a large multi-storey office, developing a new drug, etc.).

Typically all projects can be broken down into:

- Separate **activities** (tasks/jobs) – where each activity has an associated duration or **completion time** (i.e. the time from the start of the activity to its finish).

- **Precedence relationships** – which govern the order in which we may perform the activities (for example, in a project concerned with building a house the activity ‘erect all four walls’ must be finished before the activity ‘put roof on’ can start).

The problem is to bring all these activities together in a coherent fashion to complete the project.

Network analysis is a vital technique in **project management**. It enables us to take a **systematic quantitative structured approach** to the problem of managing a project through to successful completion. Moreover, as will become clear below, it has a graphical representation which means it can be understood and used by those with a less technical background. Projects typically involve many inter-related activities and one of the strengths of the techniques presented in this chapter is that they enable a project manager to adopt a systematic approach to both planning the project and to managing the project through to successful completion.

The techniques you will meet in this chapter are not a magic recipe for project management; that is to say, using them does not automatically guarantee a successful project! However it is probably true to say that attempting a project without using them does guarantee an increased probability of failure (i.e. you are more likely to fail if you neglect to use them). We hope that by the time you finish studying this chapter you also will be convinced of the value of these techniques.

**Historical background**

Two different techniques for network analysis were developed independently in the late 1950s. These were:

- **PERT** (for Program Evaluation and Review Technique)
- **CPM** (for Critical Path Management).

PERT was developed to aid the US Navy in the planning and control of its Polaris missile project. This was a project to build a strategic weapons system, namely the first submarine-launched intercontinental ballistic missile, at the time of the Cold War between the USA and Russia. Hence there was a strategic emphasis on completing the Polaris project as quickly as possible; cost was not an issue. However, no one had ever built a submarine-launched intercontinental ballistic missile before, so dealing with uncertainty was a key issue. PERT has the ability to cope with uncertain activity completion times (e.g. for a particular activity the most
likely completion time is four weeks but it could be any time between three weeks and eight weeks).

CPM was developed as a result of a joint effort by the DuPont Company and Remington Rand Univac. As these were commercial companies, cost was an issue (unlike the Polaris project considered above). In CPM the emphasis is on the trade-off between the cost of the project and its overall completion time (e.g. for certain activities it may be possible to decrease their completion times by spending more money. How does this affect the overall completion time of the project?)

Modern commercial software packages tend to blur the distinction between PERT and CPM and include options for uncertain activity completion times and project completion time/project cost trade-off analysis. Note here that many such packages exist for doing network analysis.

There is no clear terminology in the literature and you will see this area referred to by the phrases: network analysis, PERT, CPM, PERT/CPM, critical path analysis and project planning.

**Example**

We will illustrate network analysis with reference to the following example: suppose that we are going to carry out a minor redesign of a product and its associated packaging. We intend to test market this redesigned product and then revise it in the light of the test market results, finally presenting the results to the Board of the company.

The key question is:

How long will it take to complete this project?

After much thought we have identified the following list of separate activities together with their associated completion times (assumed to be known with certainty).

<table>
<thead>
<tr>
<th>Activity number</th>
<th>Activity</th>
<th>Completion time (weeks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Redesign product</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>Redesign packaging</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Order and receive components for redesigned product</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>Order and receive material for redesigned packaging</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>Assemble products</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>Make up packaging</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>Package redesigned product</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>Test market redesigned product</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>Revise redesigned product</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>Revise redesigned packaging</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>Present results to the Board</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 3.1**

Activity

Think of a small project, either at work or at home (for example, painting a room). List on a piece of paper the activities associated with this project and their associated completion times.
Aside from this list of activities we must also prepare a list of precedence relationships indicating activities which, because of the logic of the situation, must be finished before other activities can start (e.g. in the above list Activity 1 must be finished before Activity 3 can start).

It is important to note that, for clarity, we try to keep this list to a minimum by specifying only immediate relationships: that is, relationships involving activities that ‘occur near to each other in time’.

For example, it is plain that Activity 1 must be finished before Activity 9 can start but these two activities can hardly be said to have an immediate relationship (since many other activities after Activity 1 need to be finished before we can start Activity 9).

Activities 8 and 9 would be examples of activities that have an immediate relationship (Activity 8 must be finished before Activity 9 can start).

Note here that specifying non-immediate relationships merely complicates the calculations that need to be done – it does not affect the final result. Note too that, in the real world, the consequences of missing out precedence relationships are much more serious than the consequences of including unnecessary (non-immediate) relationships.

Again, after much thought (and aided by the fact that we listed the activities in a logical/chronological order), we come up with the following list of immediate precedence relationships.

<table>
<thead>
<tr>
<th>Activity number</th>
<th>Activity number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>must be finished before</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5, 6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>9, 10</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 3.2

The key to constructing this table is, for each activity in turn, to ask the question:

‘What activities must be finished before this activity can start?’

Note here that:

- Activities 1 and 2 do not appear in the right hand column of the above table. This is because there are no activities which must finish before they can start (i.e. both Activities 1 and 2 can start immediately).
- Two activities (5 and 6) must be finished before Activity 7 can start.
- It is plain from this table that non-immediate precedence relationships (e.g. ‘Activity 1 must be finished before Activity 9 can start’) need not be included in the list since they can be deduced from the relationships already in the list.

Activity

For the project you thought of previously construct on a piece of paper a list of precedence relationships.
Once we have completed our list of activities and our list of precedence relationships we combine them into a diagram (called a network – which is where the name network analysis comes from).

**Network construction**

**Activity/Reading**

For this section read Anderson, Chapter 9, section 9.1.

In the network shown below, each node (circle) represents an activity and is labelled with the activity number and the associated completion time (shown in brackets after the activity number).

![Network Diagram](image)

**Figure 3.1**

This network is an activity on node (AON) network.

In constructing the network we:

• draw a node for each activity
• add an arrow from (activity) node i to (activity) node j if Activity i must be finished before Activity j can start (Activity i precedes Activity j).

Note here that all arcs have arrows attached to them (indicating the direction the project is flowing in).

One tip that I find useful in drawing such diagrams is to structure the positioning of the nodes (activities) so that the activities at the start of the project are at the left, the activities at the end of the project at the right, and the project ‘flows’ from left to right in a natural fashion.

**Note here one key point, the above network diagram assumes that activities not linked by precedence relationships can take place simultaneously (e.g. at the start of the project we could be doing Activity 1 at the same time as we are doing Activity 2).**

Essentially the above diagram is not needed for a computer – a computer can cope very well (indeed better) with just the list of activities and their precedence relationships we had before. The above diagram is intended for people.

Consider what might happen in a large project – perhaps many thousands or tens of thousands of activities and their associated precedence relationships.

Do you think it would be possible to list those out without making any errors? Obviously not – so how can we spot errors? Looking at long lists in an attempt to spot errors is just hopeless. With a little practice it becomes easy to look at diagrams such as that shown above and interpret them and spot any errors in the specification of the activities and their associated precedence relationships.
Once having drawn the network it is a relatively easy matter to analyse it to find the critical path.

Below we repeat the network diagram for the problem we were considering before. However, note that we have now added a dummy activity (12) with a completion time of zero to represent the end of the project. This just makes the calculations we have to do easier to follow.

![Network Diagram](image)

**Figure 3.2**

**Drawing the network**

Students frequently find difficulty in drawing a correct network. Be clear here, there are two different ways to draw a network, activity on node (AON) which we have used above and activity on arc (AOA) which we have not presented in this chapter.

Our strong advice here is: activity on node is easier than activity on arc.

The structured way to draw a network diagram is:

1. Take each of the precedence relationships in turn and include them in the diagram.
2. If there are any time lags involved in the network (e.g. such as might be encountered in an examination question) then for each time lag:
   - add a dummy activity
   - connect the dummy activity to the activities involved in the time lag
3. Check that all the activities are included in the network.
4. Include a dummy **start node** if the network needs one:
   - all activities with no incoming arc are connected to the dummy start node.
5. Include a dummy **end node** if the network needs one:
   - all activities with no outgoing arc are connected to the dummy end node.

**Earliest start time calculation**

In order to analyse this network we first calculate, for each node (activity) in the network, the **earliest start time** for that activity such that all preceding activities have been finished. We do this below.
Let $E_i$ represent the **earliest start time for activity $i$ such that all its preceding activities have been finished**. We calculate the values of the $E_i$ ($i = 1, 2, ..., 12$) by going forward, from left to right, in the network diagram. To ease the notation let $T_i$ be the activity completion time associated with activity $i$ (e.g. $T_5 = 4$). Then the $E_i$ are given by:

\[
\begin{align*}
E_1 &= 0 \text{ (assuming we start at time zero)} \\
E_2 &= 0 \text{ (assuming we start at time zero)} \\
E_3 &= E_1 + T_1 = 0 + 6 = 6 \\
E_4 &= E_2 + T_2 = 0 + 2 = 2 \\
E_5 &= E_3 + T_3 = 6 + 3 = 9 \\
E_6 &= E_4 + T_4 = 2 + 2 = 4 \\
E_7 &= \max[E_5 + T_5, E_6 + T_6] = \max[9 + 4, 4 + 1] = 13 \\
E_8 &= E_7 + T_7 = 13 + 1 = 14 \\
E_9 &= E_8 + T_8 = 14 + 6 = 20 \\
E_{10} &= E_9 + T_9 = 14 + 6 = 20 \\
E_{11} &= \max[E_8 + T_8, E_{10} + T_{10}] = \max[20 + 3, 20 + 1] = 23 \\
E_{12} &= E_{11} + T_{11} = 23 + 1 = 24
\end{align*}
\]

Hence 24 (weeks) is the minimum time needed to complete **all** the activities and hence is the minimum overall project completion time.

Note here that the formal definition of the earliest start times is given by:

\[E_j = \max[E_i + T_i \mid i \text{ one of the activities linked to } j \text{ by an arc from } i \text{ to } j]\]

Conceptually we can think of this earliest start time calculation as finding the length of the **longest path** in the network (consider walking from the left-hand side of the network, to the right-hand side, through the nodes, where the completion time at each node indicates how long we must wait at the node before we can move on). **However, because of the risk of error, we should always carry out the above calculation explicitly, rather than relying on the eye/brain to inspect the network to spot the longest path in the network. This inspection approach is infeasible anyway for large networks.**

As well as the minimum overall project completion time calculated above we can extract additional useful information from the network diagram by the calculation of **latest start times**. We deal with this below.

**Latest start time calculation**

Let $L_i$ represent the **latest time we can start activity $i$ and still complete the project in the minimum overall completion time**. We calculate the values of the $L_i$ ($i = 1, 2, ..., 12$) by going backward, from right to left, in the network diagram. Hence:

\[
\begin{align*}
L_{12} &= 24 \\
L_{11} &= L_{12} - T_{11} = 24 - 1 = 23 \\
L_{10} &= L_{11} - T_{10} = 23 - 1 = 22 \\
L_9 &= L_{11} - T_9 = 23 - 3 = 20 \\
L_8 &= \min[L_9 - T_8, L_{10} - T_9] = \min[20 - 6, 22 - 6] = 14 \\
L_7 &= L_8 - T_7 = 14 - 1 = 13 \\
L_6 &= L_7 - T_6 = 13 - 1 = 12 \\
L_5 &= L_7 - T_5 = 13 - 4 = 9
\end{align*}
\]
The formal definition of the latest start times is given by:

\[ L_i = \min[L_j - T_i \mid j \text{ one of the activities linked to } i \text{ by an arc from } i \text{ to } j] \]

Note that as a check that we have done both the earliest start times and latest start times calculations correctly:

- all latest start times must be \( \geq 0 \)
- at least one activity must have a latest start time of zero.

In fact using the latest start times \( L_i \) and the concept of float we can identify which activities are critical in the above network in the sense that if a critical activity takes longer than its estimated completion time the overall project completion time will increase. We deal with this below.

**Float**

As we know the earliest start time \( E_i \) and latest start time \( L_i \) for each Activity \( i \), it is clear that the amount of slack or float time \( F_i \), available is given by \( F_i = L_i - E_i \), which is the amount by which we can increase the time taken to complete Activity \( i \) without changing (increasing) the overall project completion time. Hence we can form the table below:

<table>
<thead>
<tr>
<th>Activity</th>
<th>( L_i )</th>
<th>( E_i )</th>
<th>Float ( F_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
<td>13</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>22</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>23</td>
<td>23</td>
<td>0</td>
</tr>
</tbody>
</table>

*Table 3.3*

Any activity with a float of zero is critical. Note here that, as a check, all float values should be \( \geq 0 \).

The float figures derived are also known as **total float** as in the above example a 'chain' of successive activities (in this case 2, 4 and 6) share the same float and this is common with total float.

The float value is defined, for each activity, as the amount of time that each activity can be delayed without altering (increasing) the overall project completion time. If delays occur in two or more activities then we must recalculate the project completion time. Many textbooks also refer to float by the term 'slack'.
Critical path

Activities with a slack of zero are called critical activities since they must all be completed on time to avoid increasing the overall project completion time. Hence, for this network, activities 1, 3, 5, 7, 8, 9 and 11 are the critical activities.

Activity

If any of the critical activities are delayed, will this affect the overall project completion time or not and why?

Note here that 1–3–5–7–8–9–11 constitutes a path from the initial node (node 1) to the final node (node 11) in our network diagram. This is no accident because, for any network, there will always be a path of critical activities from the initial node to the final node. Such a path is called the critical path. Note too here that the sum of the completion times for the activities on the critical path is equal to the project completion time.

Activity

Try for yourself the example given above and see if you agree with the float values presented above.

Activity

For the project you thought of previously, calculate the minimum overall project completion time. What are the critical activities?

Activity

Can there be more than one critical path? Hint: consider the same example as given above but with the completion time for Activity 10 increased to three weeks.

Checks

If you analyse a project network then there are a number of numeric checks that can be applied to check the accuracy of your calculations. These are checks in the sense that if you fail any of these checks then you must have gone wrong somewhere. Conversely, passing all the checks does not absolutely guarantee that you are correct, although it does (for example, in an examination situation) enable you to have some confidence in your calculations. These checks are:

- All activities have floats ≥ 0 – it is good practice to explicitly give a table of floats (for non-critical activities) so that you are sure that you have calculated floats for all of them.
- There exists at least one path of critical activities from the start of the project to the end of the project.
- All activities on a critical path have float zero.
- The sum of the completion times for the activities on a critical path is equal to the project completion time.
- If an activity has float zero then it must be on at least one critical path.

In addition, be clear that:

- If activity completion times increase (and the precedence relationships remain unchanged) the project completion time cannot decrease.
- If you have a change in the precedence relationships and/or two or more activity completion times change, you need to recalculate.
We should emphasise this last point here since (particularly in an examination situation) students fail to fully grasp it. So, if you have a change in the precedence relationships and/or two or more activity completion times change then:

**if**

you simply need the new project completion time you can do a partial (forward) calculation of earliest start times

**but**

if you need the new critical path(s) and/or float times you need a full recalculation (so recalculate both earliest start and latest start times).

**Excel solution**

Now examine Sheet A in the spreadsheet associated with this chapter, as shown below. You will see there that the data for the example considered above have already been entered and Excel shows the project completion time as 24 (in cell C14), just as we calculated above.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Completion time</th>
<th>Early start</th>
<th>Early finish</th>
<th>Latest start</th>
<th>Latest finish</th>
<th>Float</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>Critical</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>Critical</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2</td>
<td>9</td>
<td>2</td>
<td>10</td>
<td>8</td>
<td>8</td>
<td>Critical</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>2</td>
<td>10</td>
<td>4</td>
<td>12</td>
<td>8</td>
<td>8</td>
<td>Critical</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>1</td>
<td>9</td>
<td>13</td>
<td>13</td>
<td>0</td>
<td>0</td>
<td>Critical</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>13</td>
<td>8</td>
<td>8</td>
<td>Critical</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>1</td>
<td>13</td>
<td>14</td>
<td>13</td>
<td>14</td>
<td>0</td>
<td>Critical</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>6</td>
<td>14</td>
<td>20</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>Critical</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>8</td>
<td>20</td>
<td>23</td>
<td>23</td>
<td>0</td>
<td>0</td>
<td>Critical</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>1</td>
<td>20</td>
<td>21</td>
<td>23</td>
<td>2</td>
<td>2</td>
<td>Critical</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>1</td>
<td>23</td>
<td>24</td>
<td>23</td>
<td>0</td>
<td>0</td>
<td>Critical</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>1</td>
<td>24</td>
<td>23</td>
<td>23</td>
<td>2</td>
<td>2</td>
<td>Critical</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>24</td>
<td>24</td>
<td>23</td>
<td>23</td>
<td>2</td>
<td>2</td>
<td>Critical</td>
</tr>
<tr>
<td>14</td>
<td>Project completion time</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Spreadsheet 3.1**

You can see above that the cells in the sheet that you can change relate to the completion times for the activities. The underlying precedence relationships have been incorporated into the Excel logic and cannot be changed. Note that the sheet indicates (column H) whether a particular activity is critical or not. Float times are also given in column G.

Of course the advantage of a spreadsheet is that it can easily recalculate the situation if we change anything. For example suppose the completion time for Activity 1 increases to eight weeks – it is easy to confirm from the spreadsheet that the project completion time increases to 26 weeks (as we would suspect as Activity 1 is critical and delaying it will also delay the completion of the entire project).

Note here that we have (implicitly) assumed in calculating this figure of 24 weeks that we have sufficient resources to enable activities to be carried out simultaneously if required (e.g. Activities 1 and 2 can be carried out simultaneously).

**Activity**

Is it possible to complete the project in 23 weeks or not and why?

**Activity**

If the completion time for Activity 2 increases to five weeks, will this affect the overall project completion time or not? If it does affect the completion time what will the new completion time be?
Sheet A also lists, for each activity:

- **Earliest start**: this is the earliest possible time that an activity can begin. All immediate predecessors must be finished before an activity can start.

- **Earliest finish**: this is the earliest possible time that an activity can be finished ( = earliest start time + activity completion time).

- **Latest start**: this is the latest time that an activity can begin and not delay the completion time of the overall project. If the earliest start and latest start times are the same then the activity is critical.

- **Latest finish**: this is the latest time that an activity can be finished and not delay the completion time of the overall project ( = latest start time + activity completion time). As with start times, the activity is critical if the earliest finish and latest finish times are the same.

Note also:

a. There may be more than one critical path – in fact it often makes more sense to talk about critical activities rather than the critical path.

b. The larger the slack the less critical the activity (e.g. what would happen to the overall project completion time if the completion time for Activity 6 increased by five)?

c. Be aware that, both in the textbooks and in the literature, different ways of performing network analysis are presented – in particular:
   
   - different definitions of slack
   - different network diagrams (exchanging the role of nodes and arcs) – as previously stated there are two types of network diagram, activity on node (AON) which we have used above and activity on arc (AOA) which we have not presented in this chapter
   - different notation conventions.

### Delay activities

A situation that is often encountered is that of a delay activity. By this we mean that a specified time must elapse between the end of one activity and the start of another. Delays can also be viewed as waiting – you have to wait a certain time between the end of one activity and the start of another. Incorporating such delays into a network diagram is an easy task. Each delay adds an additional activity to the diagram. For example, consider the network diagram shown in Figure 3.1. Suppose now that that we have the following situation:

- There must be a delay of 16 weeks (or more) between the end of Activity 3 and the start of Activity 9.

Note here the use of the phrase ‘or more’. Strictly we cannot guarantee that there is a delay of **exactly** 16 weeks between the end of Activity 3 and the start of Activity 9. The precise delay that occurs depends upon the other activities in the project. However, we can impose the condition of the delay being a certain time period or longer, i.e. mathematically the delay is ≥ a specified value. For this reason we often drop the ‘or more’ when talking of delays and implicitly assume that we mean a delay of at least the period given. So here we might equally say:

- There must be a delay of 16 weeks between the end of Activity 3 and the start of Activity 9.
To incorporate this delay we simply add an activity, running from Activity 3 in Figure 3.1 to Activity 9 in Figure 3.1 with a duration (completion time) of 16 weeks.

**Activity**

Add this delay activity to Figure 3.1.

Now with this delay activity added we can carry out the same calculation for earliest and latest times as we carried out above.

**Activity**

Compute the earliest and latest times, as well as the project completion time and the float times, for the network as in Figure 3.1 but with the delay activity added.

The Excel solution when this delay activity is added can be seen in Sheet B of the spreadsheet:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Activity</td>
<td>Completion time</td>
<td>Earliest start</td>
<td>Earliest finish</td>
<td>Latest start</td>
<td>Latest finish</td>
<td>Float</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>13</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>6</td>
<td>9</td>
<td>0 Critical</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>9</td>
<td>9</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>17</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>2</td>
<td>1</td>
<td>13</td>
<td>14</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>6</td>
<td>14</td>
<td>20</td>
<td>19</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>3</td>
<td>14</td>
<td>25</td>
<td>28</td>
<td>25</td>
<td>28</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>1</td>
<td>20</td>
<td>21</td>
<td>27</td>
<td>28</td>
<td>7</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>1</td>
<td>28</td>
<td>29</td>
<td>28</td>
<td>29</td>
<td>0 Critical</td>
</tr>
<tr>
<td>13</td>
<td>Delay</td>
<td>16</td>
<td>25</td>
<td>9</td>
<td>25</td>
<td>0 Critical</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Project completion time</td>
<td>29</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Spreadsheet 3.2**

Here we can see that the project completion time is now 29 weeks with the critical path being composed of Activities 1, 3, 9, 11 and the delay activity. Note here how the delay activity can itself be critical.

**Network analysis – extensions**

There are several important extensions to the basic network analysis technique and these relate to:

- uncertain activity completion times
- project time/project cost trade-off
- resource restrictions.

We shall deal with these in turn.

**Uncertain activity completion times**

**Activity/Reading**

For this section read Anderson, Chapter 9, section 9.2.

In this extension to the basic method we give, for each activity, not a single completion time but three times:

- optimistic time ($t_1$): the completion time if all goes well
- most likely time ($t_2$): the completion time we would expect under normal conditions
- pessimistic time ($t_3$): the completion time if things go badly.
This use of three time estimates is the PERT technique.

These three times are combined into a single figure – the expected activity completion time, given by \((t_1 + 4t_2 + t_3)/6\) and – this figure is used as the activity completion time when carrying out the calculations presented before to find the project completion time and the critical activities.

Note here that this weighting of optimistic:most likely:pessimistic of \(1/6:4/6:1/6\) is essentially fixed and cannot be altered (as the underlying theory depends on these weights).

In addition, through the use of the beta and normal probability distributions, we can get an idea of how the project completion time might vary (remember we no longer know the individual activity completion times with certainty).

Essentially we can find answers to questions like:

What is the probability that:

• the project will take longer than...?
• the project will be finished by...?
• a particular activity will take longer than...?
• a particular activity will be finished by...?

For the purposes of this subject, you will not be expected to know how to answer such questions numerically, merely to be aware that such questions can be asked and answered.

To provide more insight into this extension to the basic method suppose we have an activity for which the optimistic time is two weeks, the most likely time is five weeks and the pessimistic time is 11 weeks. So we have \(t_1 = 2, t_2 = 5, t_3 = 11\). Then the expected activity completion time is given by \((t_1 + 4t_2 + t_3)/6 = 5.5\) weeks. So we will expect that this activity will take 5.5 weeks (note that fractional values are perfectly permissible, many things in life take a fraction of a week).

The beta completion time distribution for this activity, which is the probability distribution governing the completion time for this activity given these three time estimates of \(t_1 = 2, t_2 = 5, t_3 = 11\), is (approximately) as shown in Figure 3.3.

![Figure 3.3: Variation in completion time, beta distribution.](image)
In this figure the highest point on the probability curve (with probability of approximately 0.011) corresponds to the most likely time $t_\text{m}=5$. Notice how the distribution is not symmetric (for example compare the left-hand side of the distribution between times 2 and 3 and the right-hand side of the distribution between times 10 and 11. If the distribution were symmetric these would be mirror images of each other, clearly here they are not.

**Project time/project cost trade-off**

**Activity/Reading**

For this section read Anderson, Chapter 9, section 9.3.

In this extension to the basic method we assume that, for each activity, the completion time can be reduced (within limits) by spending more money on the activity. Essentially, each activity now has more than one possible completion time (depending upon how much money we are willing to spend on it).

This use of cost information is the CPM technique.

A common assumption is to say that for each activity the completion time can lie in a range with a linear relationship holding between cost and activity completion time within this range (as illustrated below).

![Activity cost vs. Activity completion time](image)

**Figure 3.4**

Reducing an activity completion time is known as ‘crashing’ the activity and, for a given project completion time, the problem is to identify which activities to crash (and by how much) so as to minimise the total cost of achieving the desired (given) project completion time. This can be done using linear programming. For the purposes of this subject you will not be expected to know how to do this, merely to be aware that this is how cost crashing is done.

**Resource restrictions**

Typically, in real-world network analysis, each activity has associated with it some resources (such as men, machinery, materials, etc). We mentioned before that, in calculating the minimum overall project completion time, we took no account of any resource restrictions. To illustrate how network analysis can be extended to deal with resource restrictions consider the activity on node network we had before in the case of certainty with respect to activity completion times, for which the network diagram is reproduced below.
The Gantt chart below, taken from Sheet D – which reflects the information shown in Sheet C, illustrates when each activity can take place. To meet the minimum project completion time of 24 weeks for the above project, critical activities must take place at fixed points in time. For non-critical activities however we have flexibility (within limits) as to precisely when these activities take place.

To remind you of the interpretation of the Gantt chart above we have (somewhat unconventionally) shown time on the vertical axis and each activity along the horizontal axis. The solid column joins the earliest start and earliest finish times for each activity.

A key point to grasp here is that in order for the project to be completed on time (here a completion time of 24 weeks) all critical activities must start at their earliest start times and finish at their earliest finish times (i.e. we have no flexibility as to when those activities occur).

Recall that for this project the critical activities are 1, 3, 5, 7, 8, 9 and 11 and the non-critical activities are 2, 4, 6 and 10.

We consider just one resource restricted problem, resource smoothing (also known as resource levelling).

Resource smoothing

Suppose now that we have just one resource (people) associated with each activity and that the number of people required is:

- two for Activity 1
- one person for all the other activities (Activities 2 to 11 inclusive)
The spreadsheet, sheet C, is shown (in part) below.

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
<td>H</td>
<td>I</td>
<td>J</td>
</tr>
<tr>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>1</td>
<td>Activity</td>
<td>Completion time</td>
<td>Resource</td>
<td>Suggested start time</td>
<td>Earliest start</td>
<td>Earliest finish</td>
<td>Latest start</td>
<td>Latest finish</td>
<td>Float</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>7</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>7</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>12</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>Project completion time</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Spreadsheet 3.3

Column C in that spreadsheet gives the resource usage for each activity, column D is the suggested start time (if we wish to impose our own suggested start time for each activity). There are other cells in the spreadsheet beyond column I that contain values but these are concerned with internal calculations in order to calculate a resource profile.

Suppose now that we decide:

• we wish to meet the minimum overall project completion time of 24 weeks (hence implying that the times at which critical activities occur are fixed)
• we wish to start all non-critical activities at their earliest possible start times

then in the light of these decisions what does the plot (profile) of resource usage (number of people used) against time look like?

Using our spreadsheet from Sheet E the plot of resource usage against time is:

Figure 3.7

The peak of the resource profile is associated with the start of the project, when Activity 1 requires two people and the other activities, which are being performed simultaneously with Activity 1, require one person.

A key question is:

**What resource usage profile would you have most liked to have seen here?**

Clearly the ideal is a constant profile of resource against time (i.e. a constant usage of resource over time). This is because variations from a constant (straight line) profile most likely cost us extra money – either in terms of hiring extra resource to cover peaks in the resource profile, or in terms of unutilised resources when we have troughs in the resource
profile. Although it may not be possible to achieve an ideal resource profile we should keep this ideal in mind.

Now another key question is:

**Is the resource profile that we see fixed or can it be altered?**

The simple answer is that the resource profile is not fixed. It can be altered. Simply put, changing the start time for an activity changes the resource profile. This is illustrated below where we have changed the start time for Activity 2 to Time 2. It can be seen that the project is still completed in 24 weeks, but the resource profile is different.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Activity</td>
<td>Completion time</td>
<td>Resource</td>
<td>Suggested start time</td>
<td>Earliest start</td>
<td>Earliest finish</td>
<td>Latest start</td>
<td>Latest finish</td>
<td>Float</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>8</td>
<td>6</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>10</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>9</td>
<td>9</td>
<td>13</td>
<td>9</td>
<td>13</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>6</td>
<td>7</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>4</td>
<td>2</td>
<td>13</td>
<td>14</td>
<td>13</td>
<td>14</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>6</td>
<td>1</td>
<td>14</td>
<td>20</td>
<td>14</td>
<td>20</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>8</td>
<td>1</td>
<td>20</td>
<td>23</td>
<td>20</td>
<td>23</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>7</td>
<td>1</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>23</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>7</td>
<td>1</td>
<td>23</td>
<td>24</td>
<td>23</td>
<td>24</td>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Project completion time</td>
<td>24</td>
</tr>
</tbody>
</table>

**Spreadsheet 3.4**

**Figure 3.8**

So is our current usage of resource ideal? Plainly not, but what flexibility do we have? If we still wish to complete in 24 weeks we can do **nothing** with regard to the critical activities.

However, we have some choice for the non-critical activities. Recall that such activities have an associated float (slack) time. We could artificially delay starting some of these activities. If we do so the resource profile will change, maybe for the better. Indeed this is what we did implicitly above. There, in delaying the start of Activity 2 until Time 2, we still completed the project on time, but had a different resource profile. In fact for this particular example it is easy to see that delaying starting Activity 2 until Time 6 leads to a better resource profile, and is still feasible in terms of the 24 weeks completion time.

Using our spreadsheet to delay starting Activity 2 until Time 6, when Activity 1 will have finished we have:
Here we clearly have a resource profile more in line with our ideal of a constant resource usage.

It is important to note however that artificially delaying the start of non-critical activities in order to improve a resource profile is not free. Rather it costs us. Simply put, time lost by delaying the start of an activity cannot later be regained (if things do not turn out as planned) given the desired fixed completion time for the overall project.

This has illustrated the resource smoothing or resource levelling problem, which can be stated as:

- Given a fixed overall project completion time (which we know is feasible with respect to the resource constraints) 'smooth' the usage of resources over the timescale of the project (so that, for example, we do not get large changes between one week and the next in the number of people we need).

This smoothing process makes creative use of float to artificially delay activities in order to smooth resource usage.

There are two disadvantages to smoothing:

- If we have multiple resources then we may well find that smoothing one resource makes another resource less smooth, hence we have to make trade-offs between resource profiles.

- Time lost cannot be regained, once we have delayed an activity to smooth a resource profile then, if things go wrong later (e.g. some activities take longer than we planned) we cannot regain the time we have lost.
Network analysis – benefits

Network analysis is nowadays very widely used so it might be profitable to consider the benefits that using network analysis can bring to a project.

Structure

Forming the list of activities, precedence relationships and activity completion times structures thought about the project and clearly indicates the separate activities that we are going to have to undertake, their relationship to one another and how long each activity will take. Hence network analysis is useful at the planning stage of the project.

Management

Once the project has started then the basic idea is that we focus management attention on the critical activities (since if these are delayed the entire project is likely to be delayed). It is relatively easy to update the network, at regular intervals, with details of any activities that have been finished, revised activity completion times, new activities added to the network, changes in precedence relationships, etc and recalculate the overall project completion time. This gives us an important management tool for managing (controlling) the project.

Plainly it is also possible to ask (and answer) ‘what if’ questions relatively easily (e.g. what if a particular activity takes twice as long as expected – how will this affect the overall project completion time?).

It is also possible to identify activities that, at the start of the project, were non-critical but which, as the project progresses, approach the status of being critical. This enables the project manager to ‘head off’ any crisis that might be caused by suddenly finding that a previously neglected activity has gone critical.

Activity

Think about projects you have been involved with, either at work, at home, or in your community. Would these projects have benefited from applying the techniques presented in this chapter or not? Next time you have a project to do will you attempt to apply the techniques presented in this chapter or not? If not, why not?

Network analysis – state of the art

Computer packages are widely available for network analysis (including packages on PCs). Typically such packages will:

- draw network diagrams
- calculate critical activities, and the overall project completion time
- cope with uncertain activity times
- perform project time/project cost trade-off
- deal with multiple projects
- provide facilities for updating the network as the project progresses.

Problems involving thousands of activities can be easily handled on a PC.

Links to other chapters

The topics considered in this chapter do not directly link to other chapters in this subject guide. At a more general level the link between this chapter and other chapters in this subject guide is the use of a quantitative (analytic) approach to a problem.
Case studies

The case studies associated with this chapter are given below. We would encourage you to read them.

<table>
<thead>
<tr>
<th>Title</th>
<th>Anderson (page number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nokia networks</td>
<td>371</td>
</tr>
<tr>
<td>Hospital revenue bond at Seasongood &amp; Mayer</td>
<td>381</td>
</tr>
<tr>
<td>Kimberly-Clark Europe</td>
<td>391</td>
</tr>
</tbody>
</table>

A reminder of your learning outcomes

Having completed this chapter, and the Essential reading and activities, you should be able to:

- draw a network diagram
- calculate the project completion time
- calculate the earliest start time, latest start time and float time for each activity
- identify the critical activities/critical path(s)
- explain the effects of uncertain activity times and cost/time trade-offs
- explain resource smoothing
- explain the benefits of network analysis.

Sample examination questions

For Sample examination questions relating to the material presented in this chapter please visit the VLE.